Proceedings of the ASME 2013 32nd International Conference on Ocean, Offshore and Arctic Engineering
OMAE2013
June 9-14, 2013, Nantes, France

OMAE2013-11557

THREE DIMENSIONAL FULLY LOCALIZED WAVES ON ICE-COVERED OCEAN

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ABSTRACT

We have recently shown [1] that fully-localized three-dimensional wave envelopes (so-called dromions) can exist and propagate on the surface of ice-covered waters. Here we show that the inertia of the ice can play an important role in the size, direction and speed of propagation of these structures. We use multiple-scale perturbation technique to derive governing equations for the weakly nonlinear envelope of monochromatic waves propagating over the ice-covered seas. We show that the governing equations simplify to a coupled set of one equation for the envelope amplitude and one equation for the underlying mean current. This set of nonlinear equations can be further simplified to fall in the category of Davey-Stewartson equations [2]. We then use a numerical scheme initialized with the analytical dromion solution of DSI (i.e. shallow-water and surface-tension dominated regimes of Davey-Stewartson equation) to look for dromion solution of our equations. Dromions can travel over long distances and can transport mass, momentum and energy from the ice-edge deep into the solid ice-cover that can result in the ice cracking/breaking and also in posing dangers to ice-breaker ships.

INTRODUCTION

Two-dimensional solitary waves were first observed by John Scott Russell [3, 4]. About half a century later Korteweg and de Veries derived the nonlinear governing equations and found analytical form of 2D solitary waves. The profile of a two-dimensional solitary wave- similar to the one Russell observed- decays exponentially fast in all horizontal directions except along a ray. Later it was shown that governing equations for two-dimensional weakly nonlinear envelope of monochromatic waves reduces to the Nonlinear Schroedinger equation [5] and it too admits soliton solutions. Extension of KdV equation to three-dimension is obtained by Kadomtsev and Petviashivili [6] (long waves and slow transverse dependence) and that of NLS equation by Davey and Stewartson [2].

On water dominated by surface tension(i.e., Bond number$>1/3$), the KP equation (KPI) admits three dimensional fully localized structures named lumps [7, 8] and the long wave limit of DS equation (DSI) is found to admit dromions [9–12]. Both lumps and dromions are capable of propagating on water with constant speeds without changing their forms. The difference is that dromions decay exponentially in space while lumps algebraically. Also dromions form at the intersection of line-solitary mean-flow tracks and therefore their underlying structure to the leading order extends to infinity or finite boundaries.

In polar area, waves can propagate on the surface of ice-covered waters. For these waves to exist bending of the ice must be taken into account and therefore these waves are often called *flexural-gravity* waves. Many studies have been done on flexural gravity waves based on two dimensional model or/and linear wave theory [13–15]. Due to the flexural rigidity of ice, dromions can exist on water of depth much larger than that for capillary-gravity waves [1]. This study is motivated by observations of (relatively) large amplitude localized waves deep inside the icepack in polar waters. For instance 560km from the ice edge at Weddell Sea observations of breakup of an ice pack due to a series of wave packets of approximately 1m in amplitude and 18s in period have been reported [16] (also see [17] for a similar event). Three dimensional effect is believed to play a role for waves to travel so far from the ice edge and wave energy of high concentrations is believed to be responsible for the ice breaking. The characteristics and phenomena are in accordance to dromion structures.

Since the presentation of Davey-Stewartson equation, considerable work has been done in the study of its solutions. However, the analytic dromions known so far are only limited to Davey-Stewartson I equation which governs the propagation of nonlinear wave packets in the limit of long waves on water dominated by surface tension. We have recently proposed a numerical algorithm that can obtain dromion solutions for the elliptic-hyperbolic subfamily of DS equations [1]. In the Euler equations associated with nonlinear waves propagating on ice-covered water, inertia of ice sheet is usually neglected in the dynamic boundary condition in the former studies [1,15]. Here we show that the effect of the inertia on the shape, speed and direction of propagation of a dromion can be significant. We derive the governing equation for nonlinear wave packets propagating on ice-covered water including the inertia of ice into account. By applying the scheme proposed in [1] we find dromions numerically in a variety of depths much larger than that of capillary-gravity waves. The methods employed here can be simply extended to study hydroelastic dromions on water bounded by an elastic plate, e.g., large floating airports and bridges in ocean, associated with which kinds of wave body interaction problems have been studied [18–20].

Governing Equations

We consider the propagation of wave packets on ice-covered water of depth h. Flow is assumed to be incompressible, inviscid and irrotational. A Cartesian coordinate system is defined such that x-y plane rests on the interface of ice and water surface and z-axis points upward. Applying linear plate equation for thin plate with small deflection to form the dynamic boundary condition, we have the following governing equations:

$$\phi_{xx}+\phi_{yy}+\phi_{zz}=0,\ -h<z<\eta \tag{1a}$$

$$\phi_z=\eta_t+\phi_x\eta_x+\phi_y\eta_y,\ z=\eta \tag{1b}$$

$$\phi_t+\frac{1}{2}(\phi_x^2+\phi_y^2+\phi_z^2)+g\eta+H_0\nabla^4\eta+R_0\eta_{tt}=0,\ z=\eta \tag{1c}$$

$$\phi_z=0,\ z=-h \tag{1d}$$

where, ϕ is the velocity potential; η is the wave elevation; λ is the typical wave length; $\nabla^4=\partial_{xxxx}+2\partial_{xxyy}+\partial_{yyyy}$ is the biharmonic operator; $H_0=EL^3/12(1-\nu^2)\rho$ in which E is Young's modulus, L is the thickness of ice sheet, ν is the Poisson's ratio of ice, ρ is the density of water; $R_0=\rho_I L/\rho$ in which ρ_I is the density of the ice. Note that in the governing equations surface tension is neglected for the fact that its effect is trivial for waves on water of depth larger than a small boundary value(for capillary-gravity waves, the limit is less than 5mm). The results presented here are capable of recovering those from Davey and Stewartson [2] by neglecting terms related to ice.

We reset the origin on the bottom and define such transformations that all the variables are made dimensionless:

$$\phi^*=\frac{h}{a\lambda\sqrt{gh}}\phi,\ t^*=\frac{\sqrt{gh}}{\lambda}t,\ x^*=\frac{x}{\lambda}$$
$$y^*=\frac{y}{\lambda},\ z^*=\frac{z+h}{h},\ \eta^*=\frac{1}{a}\eta$$

Dropping asterisks we get:

$$\phi_{zz}+\delta^2(\phi_{xx}+\phi_{yy})=0,\ 0\le z\le 1+\varepsilon\eta \tag{2a}$$

$$\phi_z=\delta^2(\eta_t+\varepsilon\phi_x\eta_x+\varepsilon\phi_y\eta_y),\ z=1+\varepsilon\eta \tag{2b}$$

$$\phi_t+\frac{1}{2}\varepsilon(\frac{1}{\delta^2}\phi_z^2+\phi_x^2+\phi_y^2)+\eta+H\nabla^4\eta+R\eta_{tt}=0,\ z=1+\varepsilon\eta \tag{2c}$$

$$\phi_z=0,\ z=0 \tag{2d}$$

where,

$$H=\frac{H_0}{g\lambda^4},\ R=\frac{R_0 h}{\lambda^2},\ \delta=\frac{h}{\lambda},\ \varepsilon=\frac{a}{h} \tag{3}$$

Assuming $O(\varepsilon)\ll 1$, we are able to apply perturbation method to the problem and have the following perturbation expansions for η and ϕ:

$$\phi=\sum_{n=0}^{\infty}\varepsilon^n\phi_n,\ \eta=\sum_{n=0}^{\infty}\varepsilon^n\eta_n$$

To study phase velocity and group velocity, such new variables as follows are further defined:

$$\xi=x-c_pt,\ \zeta=\varepsilon(x-c_gt),\ Y=\varepsilon y,\ \tau=\varepsilon^2 t$$

where, c_p is the phase velocity and c_g is the group velocity.

Since only harmonic waves are of interest to us, we write η_n and ϕ_n as summations of harmonic modes:

$$\eta_n=\sum_{m=0}^{n+1}A_{nm}E^m+c.c.,\ \phi_n=\sum_{m=0}^{n+1}F_{nm}E^m+c.c. \tag{4}$$

where, $c.c.$ is the complex conjugate to make η_n and ϕ_n real;

$$E = exp(ik\xi),\ F_{nm} = F_{nm}(\zeta,\tau,Y,z),\ A_{nm} = A_{nm}(\zeta,\tau,Y)$$

$n = 1,2,\ldots,$ in which k is the wave number.

The governing equations in the newly defined variables become:

$$\phi_{zz} + \delta^2(\phi_{\xi\xi} + 2\varepsilon\phi_{\xi\zeta} + \varepsilon^2\phi_{\zeta\zeta} + \varepsilon^2\phi_{YY}) = 0,\ 0 \le z \le 1+\varepsilon\eta \quad (5a)$$

$$\phi_z = \delta^2[\varepsilon^2\eta_\tau - \varepsilon c_g\eta_\zeta - c_p\eta_\xi + \varepsilon(\phi_\xi + \varepsilon\phi_\zeta)(\eta_\xi + \varepsilon\eta_\zeta) + \varepsilon^3\phi_Y\eta_Y],\ z = 1+\varepsilon\eta \quad (5b)$$

$$\varepsilon^2\phi_\tau - \varepsilon c_g\phi_\zeta - c_p\phi_\xi + \frac{1}{2}\varepsilon[\frac{1}{\delta^2}\phi_z^2 + (\phi_\xi + \varepsilon\phi_\zeta)^2 + \varepsilon^2\phi_Y^2] + \eta = -H(\eta_{\xi\xi\xi\xi} + 4\varepsilon\eta_{\xi\xi\xi\zeta} + 6\varepsilon^2\eta_{\xi\xi\zeta\zeta} + 2\varepsilon^2\eta_{\xi\xi YY}) - R(-\varepsilon^2 c_p\eta_{\tau\xi} + \varepsilon^2 c_g^2\eta_{\zeta\zeta} + \varepsilon c_g c_p\eta_{\xi\zeta} - \varepsilon^2 c_p\eta_{\tau\xi} + \varepsilon c_g c_p\eta_{\xi\zeta} + c_p^2\eta_{\xi\xi}),\ z = 1+\varepsilon\eta \quad (5c)$$

$$\phi_z = 0,\ z = 0 \quad (5d)$$

We get problems in different orders of ε by expanding (5b) and (5c) about the mean water surface($z = 1$) in Taylor series and collecting terms according to the order of the samll number ε.

Leading order (ε^0) problem

Collection of terms of order ε^0 in the governing equation gives:

$$\phi_{0zz} + \delta^2\phi_{0\xi\xi} = 0,\ 0 \le z \le 1+\varepsilon\eta \quad (6a)$$

$$\phi_{0z} = -\delta^2 c_p\eta_{0\xi},\ z = 1 \quad (6b)$$

$$-c_p\phi_{0\xi} + \eta_0 + H\eta_{0\xi\xi\xi\xi} + Rc_p^2\eta_{0\xi\xi} = 0,\ z = 1 \quad (6c)$$

$$\phi_z = 0,\ z = 0 \quad (6d)$$

Substitution of (4) into the above equation, we get the expression for the phase velocity:

$$c_p^2 = \frac{(1+\tilde{H})\tanh\delta k}{\delta k + \tilde{R}\tanh\delta k}$$

where,

$$\tilde{H} = Hk^4,\ \tilde{R} = Rk^2 \quad (7)$$

From the relation of phase velocity and frequency $\omega = kc_p$, we have the dispersion relation:

$$\omega^2 = \frac{k^2(1+\tilde{H})\tanh\delta k}{\delta k + \tilde{R}\tanh\delta k}$$

where, ω is angular frequency.

Note that in (4), $\phi_0 = f_0(\zeta,Y,\tau) + F_{01}(z,\zeta,Y,\tau)E + c.c.$ in which $f_0(\zeta,Y,\tau)$ is real and only a function of ζ, τ and Y. It accounts for the potential of the underlying mean flow. In the expression of η, we set A_{00} equal to zero so that the first approximation to this problem gives purely harmonic surface wave. We denote A_{01} as A_0 for simplicity hereafter.

First order (ε^1) problem

Similarly expansion of (5) about the mean water surface $z = 0$ and collection of terms of $O(\varepsilon)$ yield:

$$\phi_{1zz} + \delta^2\phi_{1\xi\xi} + 2\delta^2\phi_{0\xi\zeta} = 0,\ 0 \le z \le 1+\varepsilon\eta \quad (8a)$$

$$\phi_{1z} + \eta_0\phi_{0zz} - \delta^2(-c_g\eta_{0\zeta} - c_p\eta_{1\xi} + \phi_{0\xi}\eta_{0\xi}) = 0,\ z = 1 \quad (8b)$$

$$-c_p\eta_0\phi_{0\xi z} + \frac{1}{2}\phi_{0\xi}^2 - c_g\phi_{0\zeta} - c_p\phi_{1\xi} + \eta_1 + \frac{1}{2\delta^2}\phi_{0z}^2 + H\eta_{1\xi\xi\xi\xi} + 4H\eta_{0\xi\xi\xi\zeta} + Rc_p^2\eta_{1\xi\xi} + 2Rc_g c_p\eta_{0\xi\zeta} = 0,\ z = 1 \quad (8c)$$

$$\phi_z = 0,\ z = 0 \quad (8d)$$

Substitution of (4) gives the expression for group velocity:

$$c_g = c_p\frac{2\delta^2k^2(1+\tilde{H}) + \delta k(1+5\tilde{H})\sinh 2\delta k + 8\tilde{R}\tilde{H}\sinh^2\delta k}{2(\delta k + \tilde{R}\tanh\delta k)(1+\tilde{H})\sinh 2\delta k} \quad (9)$$

Second order (ε^2) problem

Continuing to collect terms of $O(\varepsilon^2)$ from the expanded governing equation we have:

$$\phi_{2zz} + \delta^2\phi_{2\xi\xi} + 2\delta^2\phi_{1\xi\zeta} + \delta^2\phi_{0\zeta\zeta} + \delta^2\phi_{0YY} = 0,\ 0 \le z \le 1+\varepsilon\eta \quad (10a)$$

$$\phi_{2z} + \eta_0\phi_{1zz} + \eta_1\phi_{0zz} + \frac{1}{2}\eta_0^2\phi_{0zzz} - \delta^2[\eta_{0\tau} - c_g\eta_{1\zeta} - c_p\eta_{2\xi} + \phi_{0\xi}(\eta_{1\xi} + \eta_{0\zeta}) + \eta_{0\xi}(\phi_{0\zeta} + \phi_{1\xi} + \eta_0\phi_{0\xi z})] = 0,\ z = 1 \quad (10b)$$

$$\phi_{0\xi}\phi_{1\xi} + \phi_{0\xi}\phi_{0\zeta} + \frac{1}{\delta^2}\eta_0\phi_{0z}\phi_{0zz} + \frac{1}{\delta^2}\phi_{0z}\phi_{1z} - \frac{3}{2}\delta^2 c_p\eta_{0\xi}\phi_{0\xi}$$

$$+\eta_0\phi_{0\xi}\phi_{0\xi z} - c_g\eta_0\phi_{0\zeta z} - c_p\eta_0\phi_{1\xi z} - c_p\eta_1\phi_{0\xi z} - \frac{1}{2}c_p\eta_0^2\phi_{0\xi zz}$$
$$+\frac{3}{2}\delta^2\eta_0\eta_{0\xi}^2 - c_g\phi_{1\zeta} - c_p\phi_{2\xi} + \phi_{0\tau} + \eta_2 + H\eta_{2\xi\xi\xi\xi} + 6H\eta_{0\xi\xi\zeta\zeta}$$
$$+4H\eta_{1\xi\xi\xi\zeta} + 2H\eta_{0\xi\xi YY} + \frac{3}{2}\delta^2 H\eta_{0\xi\xi\xi\xi}\eta_{0\xi}^2 + \frac{3}{2}Rc_p^2\delta^2\eta_{0\xi\xi}\eta_{0\xi}^2$$
$$+2Rc_gc_p\eta_{1\xi\zeta} - 2Rc_p\eta_{0\tau\xi} + Rc_g^2\eta_{0\zeta\zeta} + Rc_p^2\eta_{2\xi\xi} = 0,$$
$$z = 1 \tag{10c}$$
$$\phi_z = 0,\ z = 0 \tag{10d}$$

By substitution of (4) and manipulation of related equations, we obtain Davey-Stewartson equations for flexural gravity waves:

$$(1-c_g^2)f_{0\zeta\zeta} + f_{0YY} = -\frac{1}{\sigma^2}[2\delta kc_p\sigma + (\delta^2k^2c_p^2c_g)(1-\sigma^2)]|A_0|_\zeta^2 \tag{11a}$$
$$2i\omega A_{0\tau} + \omega\omega'' A_{0\zeta\zeta} + c_pc_gA_{0YY} = 2k^2c_p[1+ \frac{\delta^2k^2c_pc_g(1-\sigma^2) - 2\tilde{R}\sigma^2}{2\sigma(\delta k + \tilde{R}\sigma)}]A_0f_{0\zeta} + \frac{k^3\delta}{2\sigma}\Gamma A_0|A_0|^2 \tag{11b}$$

where, if we define $\sigma = \tanh\delta k$, then $q\Gamma = p$ in which,

$$q = (\tilde{R}\sigma + \delta k)^3[(\tilde{R}\sigma + \delta k)(-3+12\tilde{H}) + \delta k(1+\tilde{H})(3-\sigma^2)]$$
$$p = a + b\sigma + c(1-\sigma^2) + d(1-\sigma^2)\sigma + e(1-\sigma^2)^2 + f(1-\sigma^2)^2\sigma + g(1-\sigma^2)^3$$
$$a = (52\tilde{H}^2 + 44\tilde{H} - 8)\delta^4k^4 + (48\tilde{H}^2 + 36\tilde{H} - 12)\tilde{R}^2\delta^2k^2$$
$$b = (100\tilde{H}^2 + 80\tilde{H} - 20)\tilde{R}\delta^3k^3$$
$$c = (-104\tilde{H} + 8 - 112\tilde{H}^2)\delta^4k^4 + (36 - 144\tilde{H}^2 - 108\tilde{H})\tilde{R}^2\delta^2k^2$$
$$d = (32 - 176\tilde{H} - 208\tilde{H}^2)\tilde{R}\delta^3k^3$$
$$e = (-42\tilde{H} - 63\tilde{H}^2 - 28\tilde{H}^3 - 7)\delta^4k^4 + (72\tilde{H} - 24 + 96\tilde{H}^2)\tilde{R}^2\delta^2k^2$$
$$f = (-30\tilde{H} - 3 - 24\tilde{H}^3 - 51\tilde{H}^2)\tilde{R}\delta^3k^3$$
$$g = (-2\tilde{H}^3 - 2 - 6\tilde{H}^2 - 6\tilde{H})\delta^4k^4$$

The expression of (11) is shown to conform to the form of general Davey-Stewartson equation.

Numerical Scheme

Many researchers have conducted numerical simulations to DS equations. For example, finite difference method(Crank-Nicolson scheme) is applied to elliptic-hyperbolic Davey-Stewartson equations in [21]. The scheme is tested on DSI with exact analytical dromion solutions and is shown to be capable of solving initial value problem associated with DS equations. But not much is addressed on what kind of initial data leads to dromion solutions. Split step Fourier method is applied to elliptic-hyperbolic and hyperbolic-elliptic Davey-Stewartson equations in [22]. They test the numerical scheme on DSII with exact analytic lump solution and on DSI with analytic one and 2 by 2 dromion solutions. However, since the scheme starts with the existing analytical solutions for DSI and DSII, the application of the numerical scheme to find dromions for other DS equations is limited. The numerical scheme employed here to get dromion solutions is first proposed in [1].
(11) can be further simplified to:

$$iA_{0\tau} + \lambda A_{0\zeta\zeta} + \mu A_{0YY} = (\nu_1|A_0|^2 + \nu_2 f_{0\zeta})A_0 \tag{13a}$$
$$\alpha f_{0\zeta\zeta} + f_{0YY} = -\beta|A_0|_\zeta^2 \tag{13b}$$

where,

$$\lambda = \frac{\omega''}{2},\ \mu = \frac{c_g}{2k} = \frac{\omega'}{2k} \geq 0$$
$$\nu_1 = \frac{k^3\delta}{4\sigma\omega}\Gamma,\ \nu_2 = k[1 + \frac{\delta^2k^2c_pc_g(1-\sigma^2) - 2\tilde{R}\sigma^2}{2\sigma(\delta k + \tilde{R}\sigma)}] \geq 0$$
$$\alpha = 1 - c_g^2,\ \beta = \frac{1}{\sigma^2}[2\delta kc_p\sigma + (\delta^2k^2c_p^2c_g)(1-\sigma^2)] \geq 0$$

Here we only consider elliptic-hyperbolic DS equations, i.e. $\alpha < 0$, $\lambda > 0$. If we define $v = -f_{0\zeta} + g|A_0|^2$, $g = -\beta/\alpha$, we get:

$$iA_{0\tau} + \lambda A_{0\zeta\zeta} + \mu A_{0YY} + (-\nu_1 - \nu_2 g)A_0|A_0|^2 + \nu_2 vA_0 = 0 \tag{14a}$$
$$-\alpha v_{\zeta\zeta} - v_{YY} + g|A_0|_{YY}^2 = 0 \tag{14b}$$

On the condition that $\nu_2 g + \nu_1 > 0$, the following transformations can be introduced:

$$\zeta^* = \sqrt{-\frac{1}{\mu\alpha}}\zeta,\ Y^* = \sqrt{\frac{1}{\mu}}Y,\ u^* = \frac{\sqrt{\nu_2 g + \nu_1}}{2}A_0,\ v^* = -\frac{\nu_2}{2}v$$

Dropping asterisks we get:

$$iu_\tau + pu_{\zeta\zeta} + u_{YY} - 4u|u|^2 - 2uv = 0 \tag{15a}$$
$$v_{\zeta\zeta} - v_{YY} - 4q|u|_{YY}^2 = 0 \tag{15b}$$

where,

$$p = -\frac{\lambda}{\mu\alpha} > 0,\; q = \frac{g\nu_2}{2(\nu_2 g + \nu_1)} > 0$$

According to its properties, a dromion is supposed to be stationary if put in a moving frame of reference with a constant speed the same as the dromion's. Here we re-write the equations (15) under such a moving reference frame. Its velocity vector is $c_\zeta\vec{i} + c_Y\vec{j}$ ($\vec{i}$ and $\vec{j}$ are the unit vectors along x and y directions respectively) which is independent of time. We define $\zeta^* = \zeta - c_\zeta\tau$ and $Y^* = Y - c_Y\tau$ and assume $u(\zeta^*, Y^*, \tau) = u^*(\zeta^*, Y^*)\exp(i\alpha\tau)$. Dropping asterisks, (15) becomes:

$$-\alpha u - ic_\zeta u_\zeta - ic_Y u_Y + pu_{\zeta\zeta} + u_{YY} - 4u|u|^2 - 2uv = 0 \quad (16a)$$

$$v_{\zeta\zeta} - v_{YY} - 4q|u|^2_{YY} = 0 \quad (16b)$$

in which ζ, Y are the only variables involved now.

As follows is the basic idea of this numerical scheme. The u, v are written as $u = u_0 + u_p$ and $v = v_0 + v_p$ in which u_0 and v_0 are the one-dromion solutions of DSI and u_p, v_p are deviations of dromion solutions of interest to DSI. Now u_p and v_p are the only unknowns in (16). We substitute them into (16) and apply an iterative scheme until satisfactory results are obtained. Specifically, for (16a), pseudo-spectral method is applied to calculate the derivatives with respect to ζ and Y; for (16b), we consider one of the two independent spatial variables as pseudo-time and apply the fourth-order Runge-Kutta method for time integration. Once u_p, v_p are obtained, the u_0 and v_0 are kept being updated until desired precision is obtained. We monitor the residue of these two equations by directly substituting the numerical results into (16).

Results and Discussions

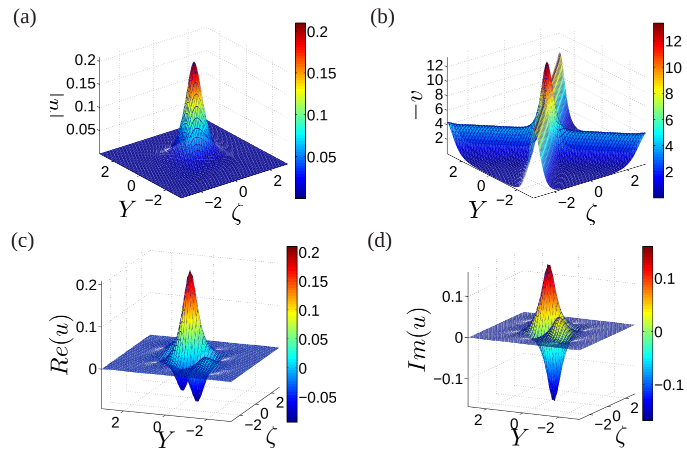

FIGURE 1: A dromion solution of flexural gravity wave to (15) for the case $kh = 10$, $\hat{R} = 0.009$ and $\hat{H} = 4.3 \times 10^{-4}$.(a) Wave amplitude $|u|$. (b) Negative velocity of the mean flow $-v$. (c),(d) Real and imaginary parts of the complex amplitude u.

From dimension analysis, we have four dimensionless groups related to the problem: kh, L/h, ρ_I/ρ, $EL^3/12(1-/\nu^2)\rho g h^4$(here ν is considered to be constant rather than variable). For simplicity, we define such notations:

$$\hat{H} = \frac{EL^3}{12(1-\nu^2)\rho g h^4},\; \hat{R} = \frac{\rho_I L}{\rho h}$$

where $\hat{R}$ stands for the ratio of inertia of ice sheet to that of water column underneath. All the coefficients in (13) can be expressed in these four independent dimensionless variables. We present a specific case that $kh = 10$, $\hat{H} = 4.3 \times 10^{-4}$. Corresponding to (16), $p = 1.50$ and $q = 0.083$(while $p = q = 1$ corresponds to DSI). Error set to 1×10^{-8}, we get the dromion solutions as in Figure 1. The velocity vector of the moving reference frame is $5.81\vec{i} + 6.59\vec{j}$ and $\alpha = 28.88$. A variety of water depths have been tested to admit such dromions.

To understand the role of inertia plays on dromions, we continue to plot the comparison of the solutions with and without taking inertia into account(by setting $\hat{R} = 0$). Letting $kh = 10$, $\hat{H} = 4.3 \times 10^{-4}$ we plot the two coefficients p and q separately for the two cases in Figure 2 for comparison. It is shown that the values of p and q in (15), when the inertia is neglected, can be much smaller. As $\hat{R}$ grows, the difference tends to increase. For the case that $k = 0.05m^{-1}$, $h = 200$m and $L = 2$m, we compare the differences of the central vertical sections of dromions in the two cases.

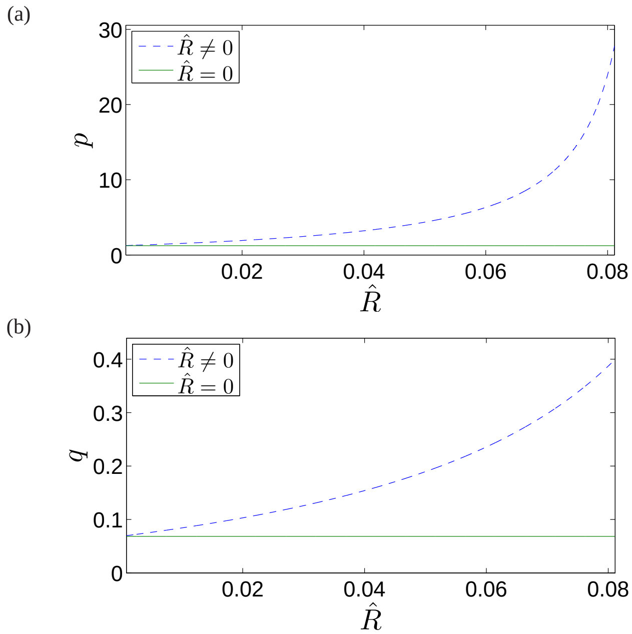

FIGURE 2: Comparison of coefficients p and q in (15) with/without taking inertia into account for the case $kh = 10$, $\hat{H} = 4.3 \times 10^{-4}$.(a) Coefficient p vs. $\hat{R}$. (b) Coefficient q vs. $\hat{R}$.

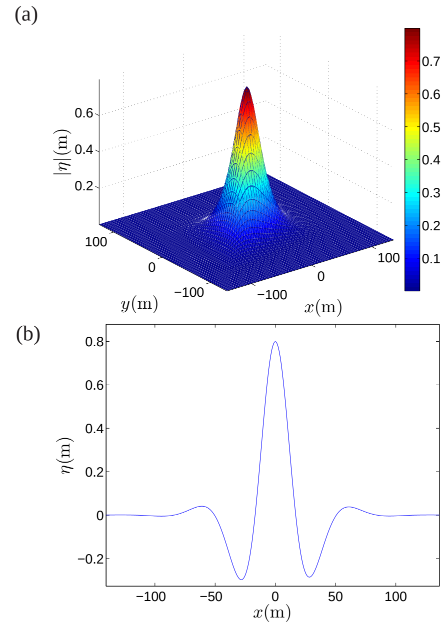

FIGURE 4: Plots in real physical space for the case $\varepsilon = 0.1$, $k = 0.05\text{m}^{-1}$, $h = 200$m and $L = 2$m; values of physical parameters of ice used are: the density of ice $\rho_I = 0.92 \times 10^3 \text{kg/m}^3$, gravitational acceleration $g = 9.8\text{m/s}^2$, Young's modulus $E = 9 \times 10^9$pa, Poisson's ratio $\nu = 0.33$. (a) Amplitude of the wave packet. (b)Wave profile at $t = 0$.

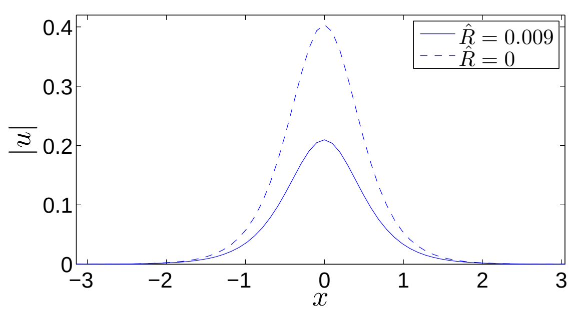

FIGURE 3: Comparison of amplitudes of dromions with/without taking inertia into account by plotting their central vertical sections at $y = 0$, the values of the specified parameters are $kh = 10$, $\hat{H} = 4.3 \times 10^{-4}$.

It is shown in Figure 3 that when the inertia of ice sheet is neglected in the computation the error of the wave amplitude is approximately 100%. For $\hat{R} = 0$, we get the velocity vector of propagation $5.33\vec{i} + 6.50\vec{j}$ which is 8.4% smaller in amplitude from the case with effect of inertia.

Since all the variables we deal with in the numerical scheme are transformed from the dimensional ones which have clearer physical meaning. By applying the inverse transform, we get the actual amplitude of the wave elevation and the velocity of the mean flow in real physical space as in Figure 4 and Figure 5.

As is mentioned previously, dromions can propagate with constant speeds without changing their forms. We are interested in if dromions on ice-covered water can preserve this property when slightly being disturbed by the ocean environment, i.e., if they can propagate with stability. A similar study for the stability of dromion solutions to DSI can be found in [23]. Their numerical results show that dromions to DSI are stable when subject to small perturbations. Here computer errors are set as the initial perturbations. we first check the conserved quantity $I = \int |u|^2 d\zeta dY$.Two time steps 5×10^{-5} and 2.5×10^{-5} are used to monitor the convergence of the results. As is shown in Figure 6, the fluctuation $|I - I_0|/I_0$ where I_0 is the initial value, after 10^5 steps, keeps at order of 10^{-6}. For the fluctuation of the maximum amplitude $|u - u_0|/|u_0|$ where u_0 is the initial complex amplitude, the error keeps around 10^{-6} as well. These differences are acceptable given that the computer errors are inevitable. Thus we can conclude that these dromions possess Lyapunov stability.

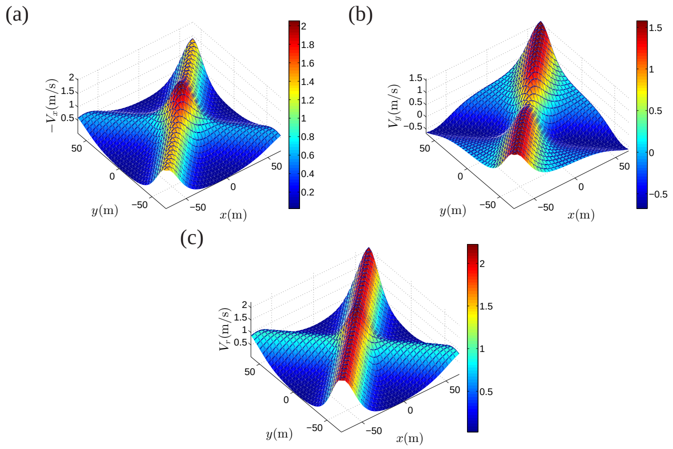

FIGURE 5: Velocity of the mean flow for the case $\varepsilon = 0.1$, $k = 0.05\text{m}^{-1}$, $h = 200\text{m}$ and $L = 2\text{m}$. (a) X-component of the velocity. (b) Y-component of the velocity. (c) Resultant velocity.

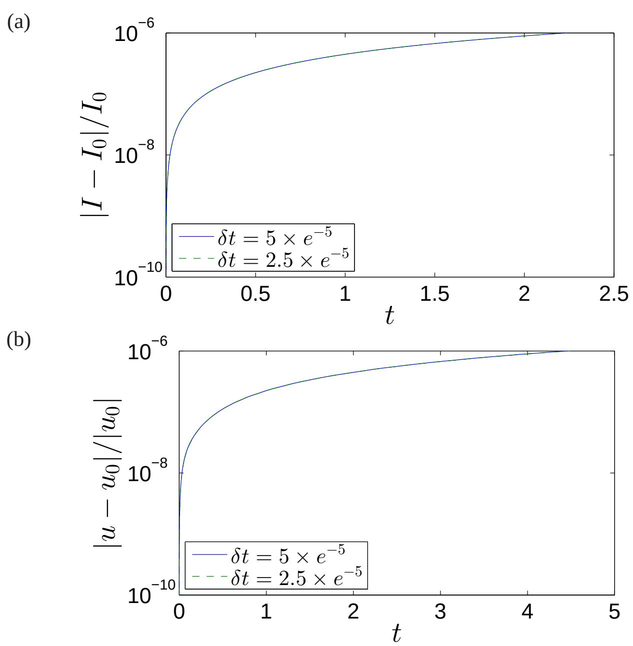

FIGURE 6: Stability study on the dromion. (a) Evolution of quantity $|I - I_0|/I_0$ over time, where $I = \int |u|^2 d\zeta dY$. (b) Evolution of quantity $|u - u_0|/|u_0|$ over time, where u_0 is the initial amplitude of the dromion.

Conclusions

Here we derive the governing equations for nonlinear wave packets on ice-covered water by multiple-scale perturbation technique and show that they conform to the form of Davey-Stewartson system. Through an iterative numerical scheme combined with pseudo-spectral method and Runge-Kutta method we obtain dromions for flexural gravity waves in much larger depth than gravity-capillary waves. We show inertia of ice sheet plays a significant role on the dromions and negligence of it can cause a large error. By applying perturbations at order of computer error, we prove that the dromions of flexural gravity waves can propagate with stability.

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