Helmholtz Resonance Mode for Wave Energy Extraction

Yalda Sadaat#1, Nelson Fernandez#2, Alexei Samimi#3, Mohammad Reza Alam*4, Mostafa Shakeri*5, Reza Ghorbani#6

#*Department of Mechanical Engineering, University of Hawai'i at Manoa*
H300, 2540 Dole st., Honolulu, HI, 96822, USA
[1] yaldas@hawaii.edu

[2] nelsonif@hawaii.edu

[3] asamimi@hawaii.edu

[6]rezag@hawaii.edu

**Department of Mechanical Engineering, University of California at Berkeley*
6111 Etcheverry Hall, University of California at Berkeley, Berkeley, California, 94720, USA
[4]reza.alam@berkeley.edu

[5]shakeri@berkeley.edu

A. *Abstract*

This study examines the novel concept of extracting wave energy at Helmholtz resonance. The device includes a basin with horizontal cross-sectional area A_0 that is connected to the sea by a channel of width B and length L, where the maximum water depth is H. The geometry of the device causes an oscillating fluid within the channel with the Helmholtz frequency of $\sigma_H^2=gHB/A_0L$ while the strait's length L, as well as the basin's length scale $A_0^{1/2}$, are much smaller than the incoming wave's wavelength. In this article, we examined the relation of above frequency to the device's geometry in both 1/25th and 1/7th scaled models at wave tank. Additionally, the sensitivity of the device in the 1/25th scale was explored where the angle of the winglets at entrance were varied to determine their effects in resonance. It was shown that the steepness of the angle of the winglets augments the higher orders of resonance in the device, thereby degrading the overall performance of the system. More over in the 1/7th scale the precision of the device to the angle between incoming waves and the device was probed through changing the device's axis, it was found that the axis changing have a negative effect in the wave amplitude at the basin and as a result on performance of the WEC.

Keywords: **Helmholtz resonance, wave energy, experimental study, wave energy converter.**

B. *Introduction*

The extraction of energy from ocean waves possesses immense potential, and with the growing global energy crisis and the need to develop alternative, reliable and environmentally non-detrimental sources of electricity, it's crucial that we learn to exploit it.

Beginning in early 19th century France, the idea of harvesting the ocean's energy has grown to become a great frontier of the energy industry. Myriad methods of extraction have arisen, notable among them Scotland's Pelamis Wave Energy Converter [1], Oyster [2], Sweden's Lysekil Project [3], Duck wave energy converter [4], the Aqua Buoy [5], Limpet [6], Wave Dragon [7], and many others. Also, Masuda [8], Mei [9], [10], [11], Flans [12], [13], Newman [14], [15], and the other pioneers established the well theoretical methods for extracting wave energy. Through these, many studies have been aimed at attaining a better understanding of wave power, a large variety of concepts have been developed, and many contributions have been made to the field--in some cases bringing commercial success as well. However, the vast majority of high-energy coastal regions still remain untapped, and there remain many creative and novel ways of applying existing technology wave energy conversion in efficient and economical ways, and with minimal environmental impact.

The objective of this article is to introduce a new wave energy converter, deemed a Rezonator. This device is inspired by geological "toilet bowls" or tidal bowls and blowholes in nature—geological phenomena that resonate at the Helmholtz mode of the waves. The Helmholtz mode is the most energetic mode of the waves and can transmit the most energy to the device. The structure of Rezonator is simple of a relatively small size. It has no moving parts and can be constructed using a low cost material such as concrete, thus maximizing a wave energy conversion system's efficiency while minimizing the maintenance and operation costs. This device is expected to facilitate the development of high performance wave energy conversion devices, broadening the European-led wave energy industry. Fig. 1 shows one example of the toilet-bowls or tidal-bowls found in abundance along Hawaiian coastal regions. The idea is straightforward: the narrow channel, basin and connection between them are of a specific geometry such that an increase quantity of the incoming wave's energy is captured. Thus, the geometry of the Rezonator is among the most important factors considered in determining the efficiency of the coupled wave energy converter. Several studies on the geometry of the device have been conducted, however the present study examines the experimental application of tidal theory, and the results are very promising.

Fig. 1 Natural Rezonator at Hanauma Bay, Hawaii.

C. *Governing Equations*

The Helmholtz mode has been applied using theoretical methods to tidal waves in studies by Doleman and Maas [16], [17]. The present study, however, examines through experimentation the application of Helmholtz resonance to wind generated ocean waves.

A Rezonator basin is illustrated from above in Fig. 2 with maximum depth H and horizontal area A_0. It is connected to the sea by a narrow strait of width B and depth H. The area A_0 depends on depth z_0/H, where z_0 is the vertical coordinate as measured from mean- water level upwards.

Maas and Robinovich [17], [18], have shown in their tidal Helmholtz resonance model that in the case that the strait's length L, as well as the basin's length $A_0^{1/2}$, are much smaller than the tidal wavelength, λ, then the wave may be assumed to traverse the basin instantaneously. Velocity and frictional effects are negligible. The

tidal wavelength $\lambda = 2\pi c/\sigma_e$, and is determined by the lunar semi-diurnal tidal frequency $\sigma_e = 1.4\times10^{-4}$ rad s^{-1}, with the long-wave

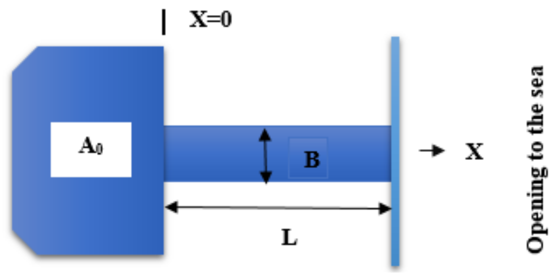

Fig. 2 Top view of the basin

speed $c = (gH)^{1/2}$ in which g denotes the acceleration due to gravity. Since the external tide elevation at sea ζ_e (σ_e , t_0) is defined as a periodic function of time t_0, the momentum equations in the basin will reduce to lowest order, rendering horizontal pressure gradients to be negligible. Thus, in the case that the breadth of the basin is much greater than the width of the channel, the uniform Helmholtz modes are the dominant modes in the basin. In addition, the tide's state in the basin can be explained by a single time dependent parameter, such as the surface elevation, ζ_i , or the volume. The excess of the water volume contained in the basin such that is:

$$V_0 \equiv \int_0^{\zeta i} A_0(z_0/H)dz_0$$

where A_0 is the basin's surface area at $z_0 = 0$, and z_0/H is the nondimentional depth of water. The change in the excess volume is a mass flux function:

$$\frac{dV_0}{dt_0} = -U(H+\zeta_0)B$$

where u(x; t_0) is the depth-averaged flow and ζ_0 (x; t_0) is the surface elevation within the strait. The flow through the strait, as derived from momentum equation, is:

$$\frac{\partial U}{\partial t} = -\frac{\partial}{\partial x}(g\zeta_{0+}\frac{1}{2}U^2) - \frac{K}{\zeta_0+H}|U|U$$

where Coriolis effects are ignored and *k* is a bed stress coefficient (k ≈ 0.0025).

It was assumed that the flow is uniform downstream of the channel where the width increases due to the basin, and the elevation decreases linearly over the length of the channel. It was also assumed that the change in surface elevation is much smaller than the depth of our system. Integrating the flow equation along the length of the channel yields:

$$L\frac{dU}{dt_0}=g(\zeta_i-\zeta_0)+\frac{1}{2}(U_i^2-U_e^2)-L\frac{K}{H}|U|U$$

(Note that the subscript "i" refers to a variable's value in the basin's interior whereas the subscript "e" refers to the exterior of the basin). After several simplifications the excess volume changes in the basin can be calculated as: $\frac{dV_0}{dt_0}$= - UHB.

There is a dynamic pressure difference between outflow and inflow, with the inflow proportional to the channel flow Ui = (1−δ)U and the Ue = U. Thus, for small δ we can rewrite (Ui_2-Ue_2) as ($-\delta U^2$) or - δ $|U|$U. This pressure drop between inflow and outflow, the bottom friction in the channel, and wave's radiation during the water oscillation in the basin all lead to a damping process in the Helmholtz oscillator. Due to this damping process, the length of the channel is effectively increased, as the water excess outside the strait takes part in the oscillation. Thus, the effective length becomes

$$L_E=L-\frac{B}{\pi}\left[ln(\frac{\pi B}{\lambda})+\Gamma-\frac{3}{2}\right]$$

In which Γ =.5772 signifies Euler's constant. Hence the total oscillating fluid's mass increases and therefore the Helmholtz frequency decreases.

In a non-dimensionalized formula for Helmholtz frequency, we assume Z_0 = Hz, ζ_i = Hζ, A_0 (Z_0/H) = A_0(A(z)), V_0 =A_0HV, dimensionless excess volume is

$$V=\int_0^{\zeta} A(z)dz, \quad \zeta e = HZe, \quad \text{and } t_0 = t/\sigma_H.$$

Thus the tidal Helmholtz frequency is characterized by:

$$\sigma_H^2=gHB/A_0LE.$$

Considering the scales and eliminating u by combining with the linear damping term, the Helmholtz resonator due to excess volume V(t) is derived such that

$$\frac{d^2V}{dt^2}+\zeta(V)=Z_e(\sigma_t)-r\frac{dV}{dt}-\gamma\left|\frac{dV}{dt}\right|\frac{dV}{dt}.$$

Here, r = σB/2L denotes the radiation damping coefficient, and γ = (δ/L + k/H)A_0/B signifies the aggregate of quadratic damping coefficients. The forcing parameters and damping parameters r and γ are independent quantities. Also, ζ(V) is the restoring term and it is the function of the excess volume. Z_e=$F\cos\sigma t$ is the forcing by the external wave and is characterized by its amplitude F and scaled frequency σ = σ_e/σ_H.

D. *Experimental Setup and Results*

The experiments successfully validate the theoretical tidal model above toward ocean wave resonance, with just a few modifications: The experimental design is geometrically scaled based on the Froude number. The geometric factor, λg, of 25 and 7 were applied to the Helmholtz frequency, σ_H, which were scaled by $\frac{1}{\sqrt{\lambda g}}$. Thus, the proposed device is examined in wave tank with 1/25th and 1/7th scales. We assumed that the root square of basin's area is much smaller than the wavelength of incoming waves. Also we assume the incoming wave's velocity is calculated by time-average power of the wave. The surface elevation for a progressive periodic wave is sinusoidal and it is a function of x and t:

$$\eta(x,t) = a\cos(kx-\omega t)$$

where a is the wave amplitude and k=2π/L denotes the wave number. The angular frequency is ω = 2π/T, where T is the wave period. The time-averaged power of the incoming wave into the device with the width of W_f is:

$$P_1 = \frac{1}{2}W_f\rho g\zeta_e{}^2Cg.$$

Here, Cg is the group velocity found by $Cg = (gH)^{0.5}$. The power at the channel is: $P_c = \frac{1}{2}\rho BH$. The power in the basin is calculated by:

$$P_2 = -\frac{1}{2}W_f\rho g\zeta_i^2\sigma_H.$$

Therefore, the ratio of power in the basin to the power of the incoming wave is:

$$\frac{P_2}{P_1}=\left(\frac{\zeta_i}{\zeta_e}\right)^2\sqrt{\frac{W_b\,B}{W_fHL}}$$

where W_b is the breadth of the square shaped basin.

The theoretical model above was tested using a 1/25th scale resonator basin in a flume at the University of Hawaii at Manoa and using a 1/7th scale at Richmond Field Station of University of California, Berkeley.

The experiment was first conducted in a flume 0.15 meters in width and sufficiently long for measurements to be taken prior reflected waves reaching the model. The basin scale was 1:25, with a channel width of 0.05 meters and a water depth of 0.2 meters. Wave heights were measured using three JFE Advantech high-sensitivity capacitive wave level sensors (ACH-600RS) with a resolution of 0.1 mm and an accuracy of ±0.03 mm. One wave gage was positioned suspended in the channel, and measured wave height therein. A second wave gave was suspended in the middle of the basin. The third was positioned at a distance greater than three water depths from the wave maker to measure fully developed incident wave height, while also spaced exactly one wavelength from the second wave gage so as to determine the phase-shift between the basin free surface and incident waves. Data was collected at a nominal 72 Hz.

A computer-controlled wave maker produced periodic, linear intermediate waves. The wave period was scaled by a geometric factor of 25, such that the target ocean periods of 4.5-7.5 seconds were scaled to 0.9-1.5 seconds. Wave height was 20% of the water depth. The channel of the resonator basin was oriented parallel to the direction of travel of incident waves. Through the use of angled winglets, the width of the channel was reduced relative to the flume. The lengths, and thus the angles, of the winglets were varied as shown in Fig. 3 to determine the effects of the angle of reduction on the incident, reflected, and transmitted waves. More detail results can be found in [19].

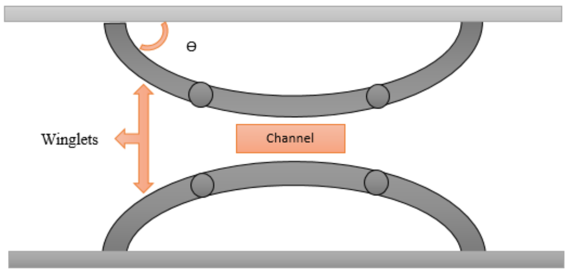

Fig 3 view of length and angle of winglets

Three sets of trials were conducted over a range of periods of winglet angles. In each case, the horizontal cross-sectional area of the basin was adjusted to match the Helmholtz resonance of the given incoming wave frequency. The first of these trial sets utilized a single winglet configuration over the full period range (0.9-1.5 seconds). The purpose of these first trials was purely to determine the validity of the theoretical model. Fig. 4 illustrates the correlation between theoretical and experimental Helmholtz frequency of the Rezonator. In the second set of trials, a static period of 1.4 seconds was utilized over the full the full range of winglet angles. These angles (Θ) were measured relative to the flume wall and ranged from 60° to 25°. It was found that lower reduction angels corresponded to a reduction of higher harmonics, and allowed for more energy to be transmitted to the basin. The third and final trial set explored the effect of a one-way gate added to the mouth of the channel. The gate allowed incident waves to enter the basin normally, while forcing the exit flow to leave the basin below the water's surface. This significantly increased the ratio of basin wave amplitude to incoming wave amplitude, demonstrating a reduction of friction damping in the channel. Exit flow velocity was augmented, suggesting performance gains with the implementation of a turbine generator.

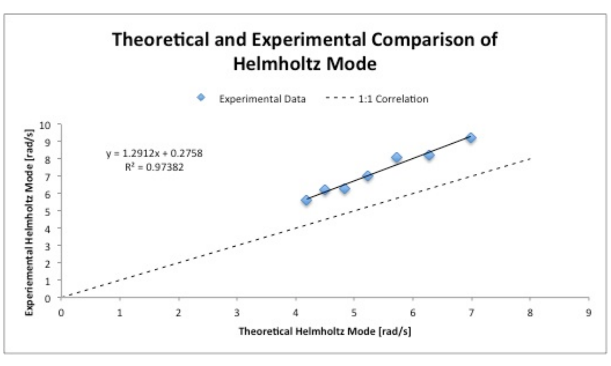

Fig. 4 Theoretical and Experimental Comparison of Helmholtz Mode.

In the 1/7th scale setup, the flume was 2.4 meters wide, 68 meters long, and 2 meters high, with a water depth of .75 meter, and was equipped with a piston actuator driven wave maker at one end.

The Rezonator was constructed using plywood panels on a frame of aluminum extrusions, as shown in Fig. 5. Dimensions were approximately 1 meter wide by 1 meter tall by 1.3 meters long. The channel width was 0.3 meters and the winglets were angled 30 degrees relative to the WEC's internal axis direction. This ideal winglet angle was determined in 1/25th scale experiments. The horizontal cross-sectional area of the basin was chosen to match the Helmholtz resonance of the basin to the incoming wave frequency.

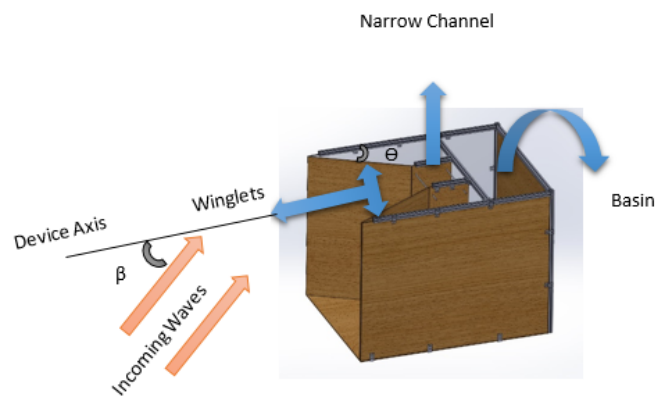

Fig. 5 The experimental device with 1/7th scale.

Two capacitive wave gauges were used to record the wave heights. The first wave gauge was positioned at distances greater than three water depths from the wave maker to measure the fully developed incident waves, and maintained a distance of one wavelength from the second wave gauge inside the middle of basin to determine the phase shift between incident waves and the basin free surface.

The periods tested were scaled using a geometric factor of 7 such that the target ocean period range of 4.5-7.5 seconds was scaled to 1.7-2.84 seconds in the experiments. Testing was primarily done with a 2.5 second period, as the basin was built to resonate at this frequency.

In addition to validating the theoretical model as was done in the first experiment, the basin was rotated by 15 degree increments from 0 degrees to 45 degrees relative to the flume wall in order to ascertain the effect of angle of incidence on internal wave height. As expected, 0 degrees produced the greatest basin wave height. Fig. 6 shows the amplitude of the incoming wave and the basin at the resonance frequency.

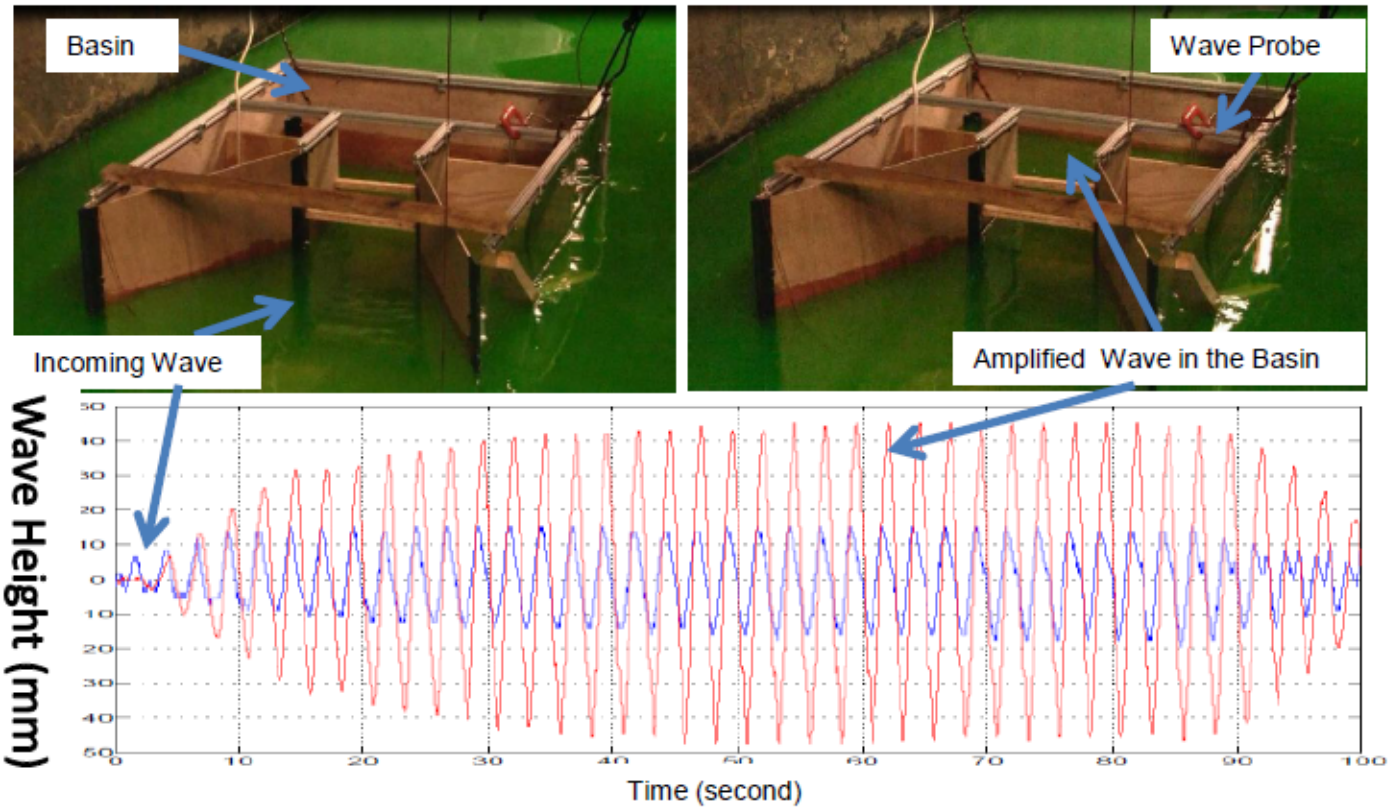

Fig. 6 1/7th scale model of Rezonator with response plot.

For the 1/7th scale experiment, the ratio of power in the basin to the incoming wave power [equation 15] reduces to:

$\frac{P_2}{P_1} = .5\left(\frac{\zeta_i}{\zeta_e}\right)^2 = 4.5$, in relative agreement with measured data. It can also be seen that the power being added to the Rezonator basin decreases from 100% at the first wave to 0 percent at the 18th wave for the steady state response. This is due to radiation and friction damping of the resonator.

The comparison of power ratios of the basin to incoming waves with different angles between WEC and incoming waves (β in Fig. 5) with β of, 0, 15, 30 degrees are shown below in Fig. 7.

As shown in Fig. 7, the ideal power ratio occurs at 0-degrees. For this case, the energy absorption of Rezonator becomes saturated at 20 wave cycles, after which the power within the system will no longer increase. Thus the optimal energy extraction interval is between the first and twentieth incident waves. It is during this time that the power take-off (PTO) be engaged so as to avoid energy extraction in a saturated system. There are two major methods of extracting the energy from the Rezonator, as shown in Fig. 8:

1. By using a water turbine at the channel. This method is ideal when the dominant wave period is low and wave amplitude is high, leading to high velocity within the channel. PIV has been used to measure the pulsating velocity at the channel for the 1/25th scaled model. The peak velocity was 0.6 m/sec for a wave period of 0.9 seconds and amplitude of 0.02 m. This is equivalent to a 3 m/s peak channel velocity at full-scale, with an ocean wave amplitude of 0.5 m. However, the nonlinear effect of incoming wave amplitude on the channel velocity must be investigated. This type of power take off is shown as “PTO1” in the Fig. 8.
2. By using an air turbine at the basin, which is shown as “PTO2” in the Fig.8.

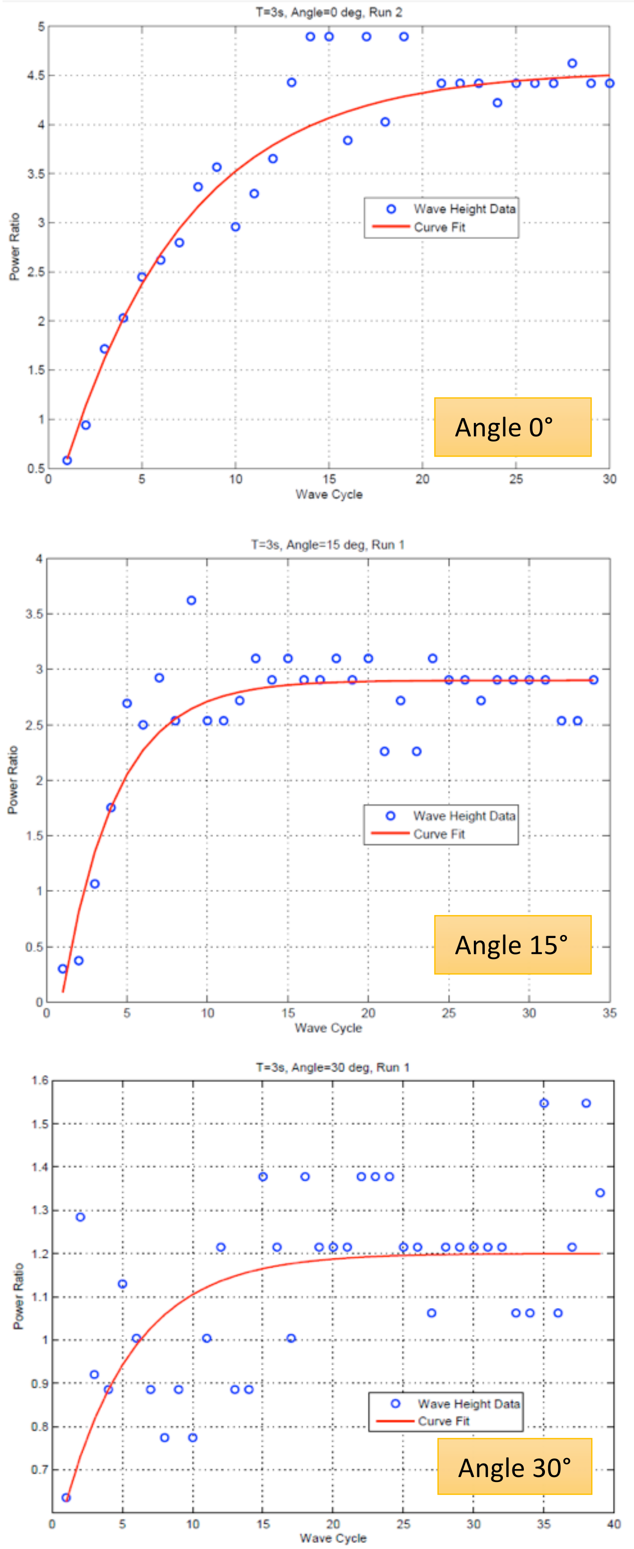

Fig. 7 power ratios of basin wave heights to incoming wave heights with different angles of incidence.

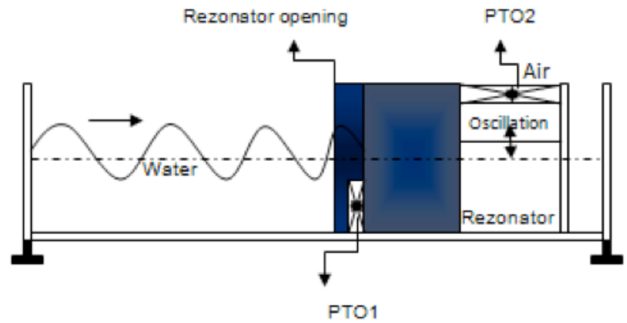

Fig. 8 Power –Take off in the Rezonator.

E. *Conclusion*

The application of the theory of Helmholtz resonance of tidal waves towards wind-driven ocean waves was explored in both 1/25th and 1/7th scale experiments, and in both cases, the theory was successfully validated. The experiments demonstrated the sensitivity of the device to the channel reduction angle, in which for the winglet angle ϴ above 30 degrees the performance of the device is declined. It can also be seen from the experiments that the power being added to the Rezonator basin decreases from 100% at the first wave to 0 percent at the 18th wave for the steady state response. This is due to radiation and friction damping of the resonator. The experiments indicate that increasing the angle between WEC and incoming waves (β) reduces the power ratio of the basin to the incoming waves.

Acknowledgment

We would like to express our gratitude to the Renewable Energy and Island Sustainability (REIS) program, which is funded in part by the US Department of Energy Grant, DE-OE0000394.

References

[1] *Yemm, R.9. (2008)."Full-scale WECs: Pelamis. in Ocean Wave Energy ". (J. Cruz, ed.), pp. 304.321.*

[2] *Aquamarine Power. Available from* http://www.aquamarinepower.com/ *(access date April 2013).*

[3] *Wave power project- Lysekil Available from* http://www.el.angstrom.uu.se/forskningsprojekt/WavePower/Lysekilsprojektet_E.html *(access date April 2013)*

[4] *Salter, S.H. (1974). "Wave Power." Nature 249:720-723.*

[5] *Wacher, A. Neilsen,K. (2010)."Mathematical and Numerical Modeling of the AquaBuOY Wave Energy Converter". Volume 2, pp. 16-33.*

[6] *Limpet,* http://www.wavegen.co.uk/what_we_offer_limpet.htm *(access date April 2013)*

[7] *(WA 2013) Wave Dragon. Available from* http://tinyurl.com/wavegen1/ *(access date April 2013).*

[8] *Masuda, Y., 1972, "Study of Wave Activated Generator and Future View as an Island Power Sources," 2nd International Ocean Development Conference.*

[9] *Mei, C.C., 1976, "Power Extraction from Water Waves," Journal of Ship Research. 20(2):63-66.*

[10] *Mei, C.C., and Newman, J.N., 1979, "Wave Power Extraction by Floating Bodies," Symposium on Ocean Wave Energy Utilization. 1-28.*

[11] *Mei, C.C., 1989," The Applied Dynamics of Ocean Surface Waves," World Scientific.*

[12] *Falnes, J., and Budal, K., 1978, "Wave-Power Conversion by Point Absorbers," Norwegian Maritime Research. 6(4):2-11.*

[13] *Falnes, J., 2007, "A Review of Wave Energy Extraction," Marines Structures. 20:185-201.*

[14] *Newman, J.N., 1977, Marine Hydrodynamics, M.I.T, Press.*

[15] *Newman, J.N., and Mei, C.C., 1979, "Wave Power Extraction by Floating Bodies," Symposium on Ocean Wave Energy Utilization., 1-28.*

[16] *Doelman, A., and Koendrink, A., and Maas, L.R.M., 2002,"Quasi-periodically forced nonlinear Helmholtz Oscillatots," Physica D: Nonlinear Phenomena.***,** *164 (1-2), pp.1-27.*

[17] *Maas, L.R.M., 1997, "On the nonlinear Helmholtz response of almost-enclosed tidal basins with sloping bottoms," J. Fluid Mech., 349, pp. 361-380.*

[18] *Rabinovich, A.B., and Shirshov, P.P., 2009, "Seiches and Harbor Oscillations," Institute of Oceanology, Russian Academy of Sciences, handbook of coastal and ocean Engineering" pp.193-214, Chap.9.*

[19] Saadat, Y., Fernandez, N., and Ghorbani, R. 2013, "The Wave Energy Converter Based On Helmholtz Mode, Inspired By nature," 32nd international conference on ocean, offshore and arctic engineering, Nantes, France.