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Sensitivity of Oceanic Rogue Waves Predictions to Measurements Uncertainties

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ABSTRACT

Here we pose and answer the question of predictability of oceanic rogue waves: "How soon can we predict a rogue wave?". We answer this question by considering, in its most general form, the governing Laplace's equation with fully nonlinear boundary conditions. We look into a large number of rogue wave events, and discovered a statistically converged predictability time-scales which is a function of 1- the energy density of the ambient ocean waves, 2- height of the anticipated rogue wave, and 3- the magnitude of uncertainties in the measurements.

INTRODUCTION

For any physical system (including oceanic waves) and in an ideal (i.e. mathematical) world, if 1- exact initial conditions of the system, 2- boundary conditions and 3- the equations governing the evolution the system are known then future can be predicted precisely. We know this is not true in physical world for a number of reasons: 1- governing equations may be sensitive to perturbations (that's why we can never forecast weather well), and 2- measurements are always affected by uncertainties. *Uncertainty propagation*, particularly within complex systems, renders the prediction non-trivial.

Of our particular interest is the predictability of oceanic rogue waves. Oceanic rogue waves are short-lived very large amplitude waves (a giant crest typically followed or preceded by a deep trough) that appear and disappear suddenly in the ocean causing damages to ships and offshore structures (Dysthe et al., 2008; Onorato et al., 2001). They are rare-enough phenomena to the extent that to date very few measured cases have been documented (Mori et al., 2002), but at the same time frequent enough that every year several incidents of damage to ships and offshore structures are attributed to them (Draper, 1964, 1971; Bruun, 1994). What mechanism(s) leads to formation of rogue waves is yet a matter of dispute, although decades of research have shed a lot of light on several aspect of their existence (Kharif, 2003). For instance, it is clear today that linear theories significantly under-estimate the frequency of occurrence of such waves (Xiao et al., 2013) and nonlinear interactions and processes play a pivotal role in the series of events leading to formation of oceanic rogue waves (Janssen et al., 2003).

With the advancement in the radar technology, accurate wave height measurement over large spatial domains is now being realized (Barale and Gade, 2008; Young et al., 1985; Dankert, 2004). This combined with advanced wave-field reconstruction techniques (Blondel et al., 2010) can provide deterministic details of the current state of the ocean (i.e. surface elevation and velocity field) at any given moment of the time with a very high accuracy. This knowledge of the ocean state that, although small but, has an inevitable uncertainty in it, is known to be (usually) good enough for the prediction of average state of the ocean in the future[1]. But an important question remains whether with this knowledge the forthcoming oceanic rogue waves can be predicted and if so how much in advance and with what accuracy?

Predictability of rogue waves is, in fact, the most

[1]The general problem of prediction of future of the ocean state is a classic challenge (Massel, 1996; Golding, 1983). Available results are, however, mostly either based on linear theories (Zhang et al., 1999; Edgar et al., 2000; Abusedra and Belmont, 2011) or phase-averaged models (WAMDI-group Hasselmann et al., 1988; Booij and Holthuijsen, 1996) that cannot obtain deterministic details of oceanic wave fields necessary for prediction of extreme events such as rogue waves.

important question to be answered in the investigation of rogue waves. Prediction, at first, needs an accurate measurement of the current state of the ocean that, as discussed above, is inevitably contaminated by uncertainties. The equations governing oceanic waves are complex because they are 1- nonlinear, 2- time dependent, and 3- problem's boundary conditions (e.g. free surface) are not known a priori (the top boundary, i.e. free surface, is in fact the *solution* to the problem). These properties make the problem of water waves propagation and interaction very complex. It is known that propagation of water waves in the ocean admits myriads of nonlinear interactions that allow significant energy transfer within the spectrum and beyond. Out of this complex soup of nonlinear interactions, a *rouge wave* emerges every once in while whose importance is well appreciated today. Due to the complex nature of different interactions in the ocean, the basic mechanism behind the formation of a rogue wave is yet a matter of dispute.

While classical research about rogue waves is mainly concerned with specific mechanism(s) involved, the manuscript at hand looks at the sensitivity of rogue waves prediction to measurements uncertainties. Physics is a science of measurements, and measurements are accompanied by uncertainties. These uncertainties, for the case of oceanic waves, evolve and propagate according to the same complex equations, and may significantly affect the predictions. The time scale beyond which effects of these uncertainties are significant is called the predictability horizon.

We discovered, by studying a large number of rogue wave events, that this predictability horizon has a statistically converged *quantitative* value, which is a function of three specific items: 1- Energy density of the ambient ocean waves (denoted by *sea state*), 2- the height of the anticipated rogue wave, and 3- the magnitude of the measurement's uncertainties.

PROBLEM FORMULATION AND APPROACH

To answer this question, we consider propagation of waves on the surface of a homogeneous, incompressible and inviscid ocean of constant depth h. Let's define a Cartesian coordinate system with the x,y-axes on the mean free surface and z-axis positive upward. Assuming the flow is irrotational, a velocity potential ϕ can be defined such that $\mathbf{u}=\nabla\phi$ where $\mathbf{u}$ is the velocity vector in the fluid domain. The governing equation (conservation of mass), and boundary conditions (momentum equation and kinematic conditions) read

$$\nabla^2\phi = 0, \qquad -h < z < \eta(\mathbf{x},t) \tag{1a}$$

$$\phi_{tt} + g\phi_z + [\partial_t + 1/2(\nabla\phi\cdot\nabla)](\nabla\phi\cdot\nabla\phi) = 0, \quad z = \eta(\mathbf{x},t) \tag{1b}$$

$$\phi_z = 0, \qquad z = -h. \tag{1c}$$

where $\eta(\mathbf{x},t) = -[\phi_t + 1/2(\nabla\phi.\nabla\phi)]/g$ is the surface elevation and g is the gravity acceleration.

Consider a broadband spectrum of propagating waves on the free surface with their energy given by a JONSWAP (Joint North Sea Wave Project) spectrum (Hasselmann et al., 1973) which is from observations in the North sea for fetch-limited waves (i.e. growing sea state) and in the absence of swells. The JONSWAP spectrum can be written in the form

$$S(\omega) = \frac{\alpha_p g^2}{\omega^5} \mathrm{e}^{\beta}\gamma^{\delta} \tag{2}$$

where $\beta = -1.25(\omega_p/\omega)^4$ with ω_p being the spectrum's peak frequency, $\alpha_p = H_s^2\omega_p^4/(16I_0(\gamma)g^2)$ with H_s being significant wave height defined as four times the standard deviation of the surface elevation, $\delta = \exp[-(\omega-\omega_p)^2/(2\omega_p^2\sigma^2)]$, and σ=0.07,0.09 for respectively $\omega \le \omega_p$ and $\omega > \omega_p$. The peak enhancement factor γ typically ranges between $1<\gamma<9$, and we choose, as is typical (Hasselmann et al., 1973; Ochi, 2005), a mean value of γ=3.3. Zeroth order moment $I_0(\gamma)$ varies in the range $0.2 < I_0(\gamma) < 0.5$ and is calculated numerically (Carter, 1982); for our application $I_0(3.3)$=0.3. Wavenumber spectrum $S(k)$ is related to frequency spectrum $S(\omega)$ via $S(k) = C_g(\omega)S(\omega)$ where $C_g(\omega)$ is the group speed, therefore, $\int S(k)\mathrm{d}k = 1/2\eta^2$ where η is the amplitude of surface waves.

For the direct simulation of evolution of a wavefield initiated by a JONSWAP spectrum we utilize a phase-resolved high-order spectral technique (Dommermuth and Yue, 1987; West et al., 1987) formulated based on Zakharov's equation (Zakharov, 1968) that can take into account a large number of wave modes (typically N=$O(1000)$) and a high order of nonlinearity (typically M=$O(10)$) in the perturbation expansion in terms of the wave steepness. The scheme has already undergone extensive convergence tests as well as validations against experimental and other numerical results (Alam et al., 2010; Alam, 2012; Liu and Yue, 1998).

To quantify the effect of uncertainty on the predictability of oceanic rogue waves a three step procedure is followed: 1- We find an initial sea state that after a specific time $t=t_r$ develops a rogue wave near

$x = x_r$, 2- To include the effect of uncertainty in the initial condition, random perturbations with a Gaussian distribution- that has a zero mean such that the overall energy of the spectrum stays the same- are added to both amplitude and phases of the initial state of the ocean, 3- The new perturbed initial condition is evolved and vicinity of $t = t_r, x = x_r$ is searched for rogue (or large) waves. This wave is compared with the rogue wave that unperturbed system predicts. For each initial condition, steps one to three are repeated for a large set of initial perturbations until converged averaged quantities are obtained.

Finding an initial broadband sea state that leads to a rogue wave at a specific moment in the future is, however, a challenge. This challenge is further highlighted in a statistical investigation where a large number of such cases are needed. To overcome this issue, we propose a technique that relies on reversibility of nonlinear governing equations of oceanic waves. Specifically, if (η, ϕ) is a solution to governing equations (1) in forward time, then $(\eta, -\phi)$ is a solution to the same set of equations in the reverse time, i.e., when t is replaced by $-t$. In a forward-time simulation of governing equation (1), if a rogue wave is observed at the time $t = t_i$ then we continue the simulation up to a final time $t_f = t_i + t_r$. At $t = t_f$ water surface elevation and potential, i.e. $\eta(\mathbf{x}, t_f)$ and $\phi(\mathbf{x}, t_f)$, are recorded. A direct simulation with initial surface elevation $\eta_0 \equiv \eta(\mathbf{x}, t_f)$ and initial potential $\phi_0 \equiv -\phi(\mathbf{x}, t_f)$ will result in a rogue wave at exactly $t = t_r$.

It is to be noted that water waves show instability as a result of a number of nonlinear mechanisms such as the well-studied Benjamin-Feir instability, or, myriad of wave-wave resonances. These instabilities initially predict a one-way (i.e. non-reversible) energy transfer. A longer-time theoretical analysis (e.g. by a multiple-scales approach (Mei and Me, 1985)) which is now backed by experimental proof (Van Simaeys et al., 2002), however, reveals that after a threshold the process reverses and eventually the initial state is recovered. Therefore, the assumption of reversibility of water waves is not violated by the one-way exponential growth of computational error. Computational results presented in this paper consistently endorse this fact by showing excellent agreement in the reverse simulation.

IMPLEMENTATION AND RESULTS

For implementation of this procedure in a specific (given) sea state, we initialize our phase-resolved spectral scheme with amplitudes and frequencies given by the JONSWAP spectrum (2), and with random phases that have a uniform distribution. Via running the direct computation for a relatively short initial time t_0 (typically $t_0 \sim O(20T_p)$ where $T_p = 2\pi/\omega_p$ is the period of the peak frequency wave in the spectrum), we search for those initial phases that lead to a rogue wave event within $0 < t_i < t_0$. This initial search is relatively fast. We have performed $O(10^4)$ initial runs for each of the sea states four and five (respectively moderate and rough seas) and have collected a database of $O(100)$ cases from those initial conditions that generate rogue waves for each sea state. Water surface of the sea state five at the time of occurrence of a rogue wave with $H_{rs} = H_r/H_s$=2.78 (H_r being the crest to trough height of the rogue wave) is shown in Figure 1a. Our computational experiments shows lower sea states do not develop rogue waves with very large values of H_{rs}. Specifically, in our database for the sea state four, Max(H_{rs})∼2.6, whereas for sea state five we recorded several rogue waves with H_{rs} >2.6.

If an initial search is successful and a rogue wave is obtained at $t_i < t_0$, we then continue the evolution up to $t_f = t_i + t_r$ where $t_r = nT_p$ with $n \sim O(10^3)$. As discussed above, a new simulation with initial conditions (η_0, ϕ_0) will lead to a rogue wave exactly at $t = t_r$. In practice, this initial condition is a result of combined measurements and reconstruction procedures, and is available with an inevitable *range of uncertainty*. To

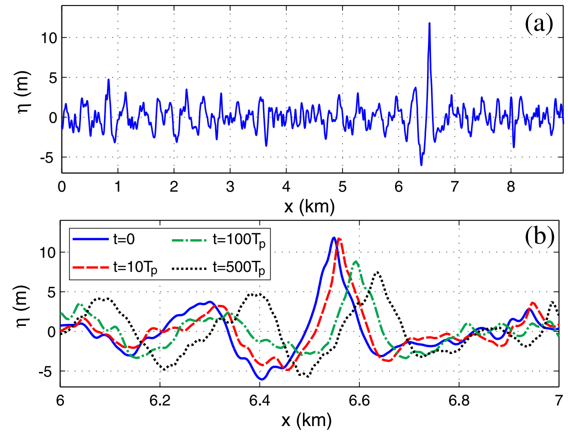

Figure 1: a. A rogue wave with H_{rs}=2.78 in the sea state five. b. Result of a E_r=10% error in the estimation of the initial state of the sea on the prediction of rogue wave. If sea state is known at t_r=10T_p before the occurance of the rogue wave then a E_r=10% error has resulted in 15% error in the predictions (red dashed line). For t=100T_p and 500T_p in advance (green dash-dotted line and black dotted line) the same error results in respectively 38% and 30% error. In the latter two cases the predicted wave is hardly a rogue wave by definition (H_{ps}=1.95,2.01).

quantify the effect of uncertainty on the predictability of a rogue wave, we add Gaussian perturbations with a zero mean and standard deviation E_r (percent) to the amplitude and phases of components of the initial condition (η_0, ϕ_0). The perturbed state of the ocean, i.e. η_{0p}, ϕ_{0p}, is then used as the initial condition and its prediction at $t = t_r$ is compared with the actual rogue wave. In practice, if the perturbed initial condition predicts a *close-enough* approximation of the height, location and time of occurrence of the rogue wave most of the prediction objective is met. To take this fact into account, we search for the highest wave in the time span of $t_r - T_p < t < t_r + T_p$ and it is further checked that this highest wave is in the $\pm\lambda_p$ vicinity of the expected rogue wave. This highest wave (with a tough to crest height of H_p) is what our predictor foresees at the vicinity of where the actual rogue wave will occur. We define $H_{ps} \equiv H_p/H_{s'}$ with $H_{s'}$ is the significant waveheight as is calculated by the predictive simulation at the time of occurrence of H_p. Clearly if E_r=0, then $H_{ps} = H_{rs}$.

Effect of a E_r=10% uncertainty in the initial condition on the prediction of the rogue wave of Figure 1a is shown in Figure 1b. At $t = 0$ original rogue wave is obtained, but if the required prediction time is longer, the effect of the initial uncertainty is more highlighted. Specifically for $t = 10T_p, 100T_p$ and $500T_p$, then respectively H_{ps}=2.41, 1.95 and 2.01 corresponding to 15%, 38% and 30% error. In fact for t=100, 500 predicted wave is hardly considered a rogue wave by the definition. Note that Figure 1b shows the effect of *one* specific set of initial perturbations on the shape of the predicted rogue wave.

To highlight the significant of nonlinearities on the prediction and to also provide a convergence test for our scheme, we compare predictions initialized by η_p, ϕ_p, for the rogue wave case presented in Figure 1a (sea state five), with E_r=0%, 5%, 10%, 15%, 20% and 25%, and by taking different orders of nonlinearity into account (Figure 2). To obtain a statistical average of the effect of uncertainty, each presented case (i.e. earch marker in Figure 2) is the average result of 19 simulations each initiated with an independent set of random perturbations. We have made sure all presented cases have a standard error of less than 2% (correspond to an error equal to $\pm$2 units in the vertical axis of Figure 2. Error bars associated with exact error of each case are not shown to avoid a crowded diagram). In each of Figure 2a-d we also consider four lead times of $t_r = nT_p$, $n = 1, 10, 100, 500$.

Linear model (M=1, Figure. 2a) under-predicts the height of the rogue wave even with E_r=0% and n=1 (i.e. zero initial disturbance and within one period of occurrence). For M=2, although for n=1 predictions are acceptable, but for later times are very much far from

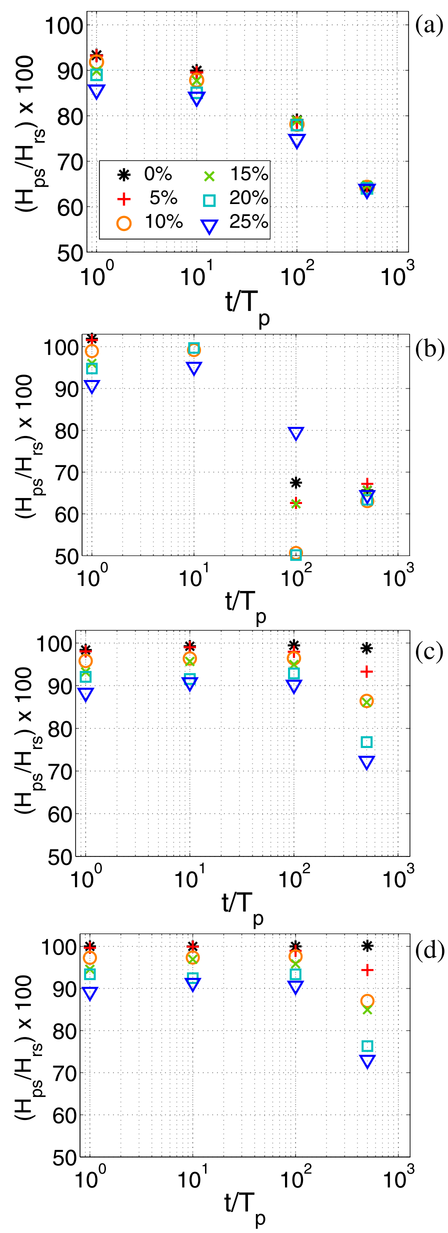

Figure 2: Effect of nonlinearities on the series of events leading to a formation of an oceanic rogue wave. Figures a-d respectively show M=1 (Linear simulation), M=2,3, and 4 (second, third and fourth order). Markers (and colors) correspond to different initial perturbations (E_r). For example, black star in Figure.a shows that the linear model (M=1), even with a zero initial uncertainty (E_r=0%), results in $\sim$5% error (i.e. $H_{ps}/H_{rs} \sim$0.95) in the prediction of the height of the rogue wave which is only $t = T_p$ ahead. From Figures c,d it is seen that convergence is achieved for M=3. Standard error of all data points are less than 2%.

converged results of M=3,4 (Figure. 2c,d). A good convergence is observed for M=3 (Figure. 2c). The general behavior observed in Figure 2c,d is qualitatively the same for all cases we investigated.

To see if there is a quantitative trend in the predictability of oceanic rogue waves, we have performed extensive numerical experiments on $\sim O(100)$ rogue waves in our database that emerge in each of the sea states four and five with normalized heights of $2 < H_{rs} < 3$. Results for $t_r = 500T_p$ are shown respectively in Figures 3a,b and c. Each marker is again an average of 19 simulations with standard error of less than 2% [2].

Error in the prediction of the height of rogue waves, as suggested by Figure 3, is a function of height of the rogue wave, sea state and, of course, degree of uncertainty. For the two sea states, the error in the prediction is higher for higher amplitude rogue waves. The rate of increase in the error is larger for sea state five (Figure. 3b) compared with sea state four (Figure. 3a). This is due to the higher nonlinearity of the ambient waves in the sea state five that further amplifies disturbances in the evolution equation.

We define predictability horizon for oceanic rogue waves at H_{ps}/H_{rs}=0.9, i.e. when rogue wave height prediction can be made with 10% accuracy[3]. This definition is based on the fact that 10% higher H_{rs} corresponds to return period of an order of magnitude longer (in years) (Leonard-Williams and Saulter, 2013). Based on this definition, a rogue wave in a sea state four is precitable $500T_P$ ahead of occurence if the uncertaintly in the sea state measurement at the current time is less than 20%. To achieve this level of predictability in a sea state 5, the uncertainty has to be less that $\sim$5%.

In practice, many offshore structures are designed today for the extreme waves of return period 10,000 years. It is crucial for these structures to know if a rogue wave with a height greater than the design value may occur at their location. If this knowledge is in hand several precautionary procedures can be carried out to minimize the damage and the potential life loss (such procedures include, for instance, shut down or relocation). Therefore a critical question is if a reliable prediction can be made. Norwegian offshore standard NORSOK (NORSOK-N-003, 2007) suggests that in the absence of more detailed information an extreme wave of H_{10000}/H_s=2.375 has an annual probability of occurrence of less than 10^{-4}. Therefore in Figures 3a-b, waves with $H_{rs} >$2.375 are larger than H_{10000}.

CONCLUDING REMARKS

In summary, here we presented a quantitative predictability horizon for oceanic rogue waves due to an uncertainty in the initial measurement of the ocean surface. This horizon in shorter in higher sea states and if the amplitude of the actual anticipated rogue wave is higher. Nonlinearity and nonlinear interactions are the major player behind the amplification of the initial uncertainty, and affect the prediction to the extent that all major features of an upcoming rogue wave may be completely lost.

The spectrum considered here is (relatively) broad. It is known, however, that narrower spectra are more amenable to instabilities, and therefore it is expected that the predictability is worse for a narrower spectrum sea state. The evolution of sea states seven and beyond involves very steep waves, wave breaking and stronger effects of viscous dissipation. Effects of wave-breaking and viscosity are expected to lower the chance of occurrence of oceanic rogue waves and hence positively contribute to the predictability horizon. Both effects can be incorporated into the spectral scheme used here by the means of semi-empirical terms (Xiao et al., 2013; Wu et al., 2006).

Sensitivity of predictions of analytical model-equations such as Nonlinear Schrödinger (Akhmediev et al., 2009b,a; Chabchoub et al., 2011; Akhmediev et al., 2010) and specific growth mechanisms such as Benjamin-Feir instability (Xiao et al., 2013; Lake et al., 1977) to the initial perturbations may provide analytical predictability horizon and worth investigation. Another important and immediate follow up question is the predictability horizon in three-dimension and how presented results here will be affected. Particularly since Benjamin-Feir instability is less determinant in three-dimension, care must be taken in the investigation and extension of current results to short-crested seas.

Rogue waves may appear not only in surface gravity wave systems but also in optical systems (Solli et al., 2007), capillary waves (Shats et al., 2010), plasma physics (Moslem et al., 2011), and in financial systems (Zhen-Ya, 2010), where techniques developed here and results obtained may be utilized toward determining the predictability.

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[2] The time $t_r = 500T_p$ corresponds to an order of $\sim$1 hour for ocean applications and is chosen because features of results and particularly effects of nonlinearity are more highlighted.

[3] The term "horizon of predictability" is used extensively, though not exclusively (López et al., 2011), in the context of chaotic dynamical systems. We would like to emphasis that, although water surface may undergo chaotic motion in cases, the use of the term "predictability horizon" here is not based on such behavior, but merely nonlinear amplification of noise by nonlinearity. Whether water surface undergoes chaotic behavior, and if so under what condition(s), is an interesting subject of research but requires a separate investigation.

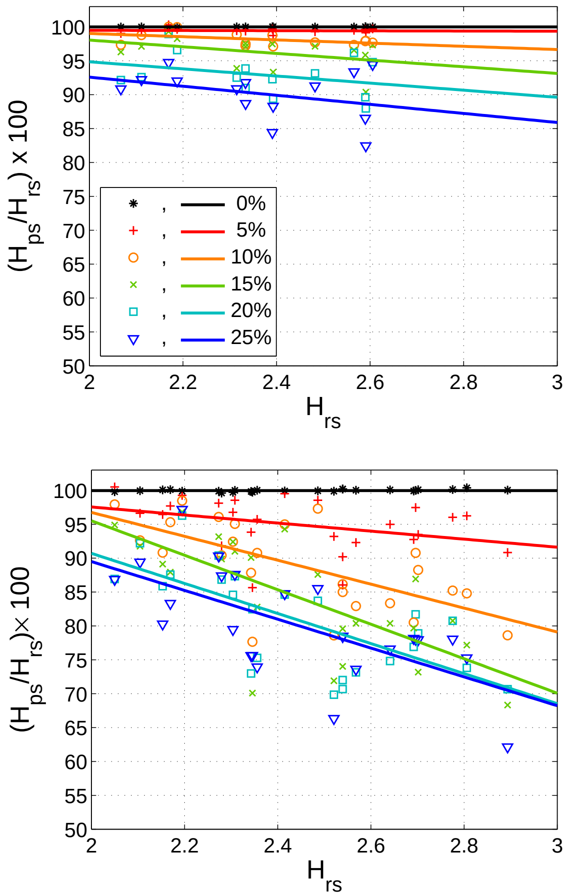

Figure 3: Predictability horizon of oceanic rogue waves. Horizontal axis (H_{rs}) shows the true amplitude of the rogue wave normalized by the significant wave height, and the vertical axis (H_{ps}/H_{rs}) shows the ratio of the amplitude predicted to the true amplitude of the rogue wave. Each marker is an average of 19 simulations initiated t =500T_p before the occurrence of a rogue wave whose relative amplitude (H_{rs}) is shown on the x-axis. Initial sea state is perturbed with Gaussian noise with standard deviations (E_r) equal to 0, i.e. no noise (black asterisks), 5% (red pluses), 10% (orange circles), 15% (green crosses), 20% (light-blue squares) and 25% (dark-blue trigangles) in all both Figures (see legend in Figure a). In the case of E_r=0 (black asterisks), as expected, rogue waves are predicted with practically zero error. For nonzero perturbations error in prediction is larger if amplitude of the rogue wave is larger, and for higher sea states (c.f. Figures a,b)

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