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**Key Points:**
- There is a predictability time scale associated with rogue wave events
- Larger rogue waves have a shorter horizon of predictability
- Rogue waves are less predictable in rougher seas (due to nonlinearities)

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# Predictability horizon of oceanic rogue waves

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**Abstract** Prediction is a central goal and a yet-unresolved challenge in the investigation of oceanic rogue waves. Here we define a horizon of predictability for oceanic rogue waves and derive, via extensive computational experiments, a statistically converged predictability time scale for these structures. We show that this time scale is a function of the sea state (i.e., severity of the ambient ocean waves), the height of the anticipated rogue wave, and the level of uncertainties in the ocean measurements. The presented predictability time scale establishes a quantitative metric on the combined temporal effects of the variety of mechanisms that together lead to the formation of a rogue wave and is crucial for the assessment of validity of rogue wave predictions, as well as for the critical evaluation of results from the widely used model equations.

## 1. Introduction

Oceanic rogue waves are short-lived very large amplitude waves (a giant crest typically followed or preceded by a deep trough) that appear and disappear suddenly in the ocean causing damages to ships and offshore structures [*Dysthe et al.*, 2008; *Onorato et al.*, 2001]. They are rare-enough phenomena that to date very few measured cases have been documented [*Mori et al.*, 2002] but at the same time frequent enough that cause annually several incidents of damage to ships and offshore structures [e.g. *Draper*, 1964, 1971; *Bruun*, 1994]. What mechanism(s) leads to formation of rogue waves is yet a matter of dispute, although decades of research have shed a lot of light on several aspects of their existence [*Kharif and Pelinovsky*, 2003]. For instance, it is clear today that linear theories significantly underestimate the frequency of occurrence of such waves [*Xiao et al.*, 2013] and nonlinear interactions and processes play a pivotal role in the series of events leading to formation of oceanic rogue waves [*Janssen*, 2003].

The central question in the investigation of rogue waves is if they can be predicted, i.e., when and where they will occur and what their features are. We are today closer than ever to answer this question owing to the advancement in the radar technology as well as surface reconstruction techniques: the state of the art radar technology can now provide accurate wave height measurement over large spatial domains [e.g. *Barale and Gade*, 2008; *Young et al.*, 1985; *Dankert*, 2004] and when combined with advanced wavefield reconstruction techniques [e.g. *Wu et al.*, 2004; *Blondel et al.*, 2010] together render deterministic (phase-resolved) details of the current state of the ocean (i.e., surface elevation and velocity field) at any given moment of the time with a very high accuracy. This knowledge of the ocean state, that although small but has an inevitable uncertainty from both measurements and reconstruction, is known to be good enough for the prediction of average state of the ocean in the future. In fact, the general problem of prediction of future of the ocean state is classic [e.g. *Massel*, 1996; *Golding*, 1983]. Available results are, however, mostly either based on linear theory [*Zhang*, 1999; *Zhang et al.*, 1999; *Edgar et al.*, 2000; *Abusedra and Belmont*, 2011] or phase-averaged models [*WAMDI Group: Hasselmann et al.*, 1988; *Booij et al.*, 1996] that cannot obtain deterministic details of oceanic wavefields necessary for the prediction of extreme events such as a rogue wave [*Pelinovsky and Kharif*, 2008]. Therefore, the important question remains whether, with the aforementioned knowledge of the current state of ocean, the forthcoming oceanic rogue waves can be predicted? And if so, how much in advance and with what accuracy?

Here we define a horizon of predictability for oceanic rogue waves in two-dimensional unidirectional broadband seas and establish, via an extensive statistical analysis, a predictability time scale as a function of measurement uncertainties. This time scale of predictability provides a quantitative metric to evaluate the range of validity of prediction efforts, and since it is obtained through primitive equations can set a standard in evaluating classical model equations (e.g., nonlinear Schrödinger equation) [cf. *Zakharov and Gelash*, 2013].

## 2. Problem Formulation and Approach

We consider propagation of waves on the surface of a homogeneous, incompressible, and inviscid ocean of constant depth $h$. Let us define a Cartesian coordinate system with the $x, y$ axis on the mean free surface and $z$ axis positive upward. Assuming the flow is irrotational, a velocity potential $\phi$ can be defined such that $\mathbf{u} = \nabla\phi$ where $\mathbf{u}$ is the velocity vector in the fluid domain. The governing equation (conservation of mass) and boundary conditions (momentum equation and kinematic conditions) read

$$\nabla^2\phi = 0, \qquad -h < z < \eta(\mathbf{x}, t) \tag{1}$$

$$\phi_{tt} + g\phi_z + [\partial_t + 1/2(\nabla\phi \cdot \nabla)](\nabla\phi \cdot \nabla\phi) = 0, \qquad z = \eta(\mathbf{x}, t) \tag{2}$$

$$\phi_z = 0, \qquad z = -h. \tag{3}$$

where $\eta(\mathbf{x}, t) = -[\phi_t + 1/2(\nabla\phi.\nabla\phi)]/g$ is the surface elevation and $g$ is the gravity acceleration.

Consider, on the ocean surface, a broadband spectrum of unidirectional propagating waves whose spectral density function, $S(\omega)$, is given by a Joint North Sea Wave Project spectrum in the form [*Hasselmann et al.*, 1973]

$$S(\omega) = \frac{\alpha_p g^2}{\omega^5} e^{\beta} \gamma^{\delta} \tag{4}$$

where $\beta = -1.25\left(\omega_p/\omega\right)^4$ with $\omega_p$ being the spectrum's peak frequency, $\alpha_p = H_s^2\omega_p^4/(16 I_0(\gamma) g^2)$ with $H_s$ being significant wave height defined as 4 times the standard deviation of the surface elevation, $\delta = \exp[-(\omega-\omega_p)^2/(2\omega_p^2\sigma^2)]$, and $\sigma = 0.07$ and 0.09 for, respectively, $\omega \le \omega_p$ and $\omega > \omega_p$. The peak enhancement factor $\gamma$ typically ranges between $1 < \gamma < 9$, and we choose, as is typical [e.g. *Hasselmann et al.*, 1973; *Carter*, 1982; *Ochi,* 2005], a mean value of $\gamma = 3.3$. Zeroth-order moment $I_0(\gamma)$ varies in the range $0.2 < I_0(\gamma) < 0.5$ and is calculated numerically [e.g. *Carter*, 1982]; for our application $I_0(3.3) = 0.3$.

For the direct simulation of evolution of a wavefield initiated by the spectrum (4), we utilize a phase-resolved high-order spectral technique [c.f. *Dommermuth and Yue*, 1987; *West et al.*, 1987] formulated based on Zakharov's equation [*Zakharov*, 1968] that can take into account a large number of wave modes (typically $N = O(1000)$) and a high order of nonlinearity (typically $M = O(10)$) in the perturbation expansion in terms of the wave steepness. The scheme has already undergone extensive convergence tests as well as validations against experimental and other numerical results [e.g. *Liu and Yue*, 1998; *Alam et al.*, 2009a, 2009b, 2010; *Toffoli et al.*, 2010; *Alam*, 2012a, 2012b, 2012c].

To quantify the effect of uncertainty on the predictability of oceanic rogue waves, a three-step procedure is followed: (1) we find a phase-resolved initial sea state [cf. *Met Office*, 2010] that after a specific time $t = t_r$ develops a rogue wave near $x = x_r$; (2) to include the effect of uncertainty in the initial condition, random perturbations with a Gaussian distribution (that has a zero mean such that the overall energy of the spectrum stays the same) are added to both amplitude and phases of the initial state of the ocean; and (3) the new perturbed initial condition is evolved and vicinity of $t = t_r, x = x_r$ is searched for rogue (or large) waves. This wave is compared with the rogue wave that the unperturbed system foresees. For each initial condition, steps one to three are repeated for a large set of initial perturbations until converged averaged quantities are obtained.

Finding an initial broadband sea state that leads to a rogue wave at a specific moment in the future is, however, a challenge. This challenge is further highlighted in a statistical investigation where a large number of such cases are needed. To overcome this issue, we propose a technique that relies on reversibility of nonlinear governing equations of oceanic waves. Specifically, if $(\eta, \phi)$ is a solution to governing equations (1)-(3) in forward time, then $(\eta, -\phi)$ is a solution to the same set of equations in the reverse time, i.e., when $t$ is replaced by $-t$. In a forward time simulation of governing equation (1)-(3), if a rogue wave is observed at the time $t = t_i$, then we continue the simulation up to a final time $t_f = t_i + t_r$. At $t = t_f$ water surface elevation and potential, i.e., $\eta(\mathbf{x}, t_f)$ and $\phi(\mathbf{x}, t_f)$, are recorded. A direct simulation with initial surface elevation $\eta_0 \equiv \eta(\mathbf{x}, t_f)$ and initial potential $\phi_0 \equiv -\phi(\mathbf{x}, t_f)$ will result in a rogue wave at exactly $t = t_r$.

It is to be noted that water waves show instability as a result of a number of nonlinear mechanisms such as the well-studied Benjamin-Feir instability or myriad of wave-wave resonances. These instabilities initially predict a one-way (i.e., nonreversible) energy transfer. A longer-time theoretical analysis (e.g., by a

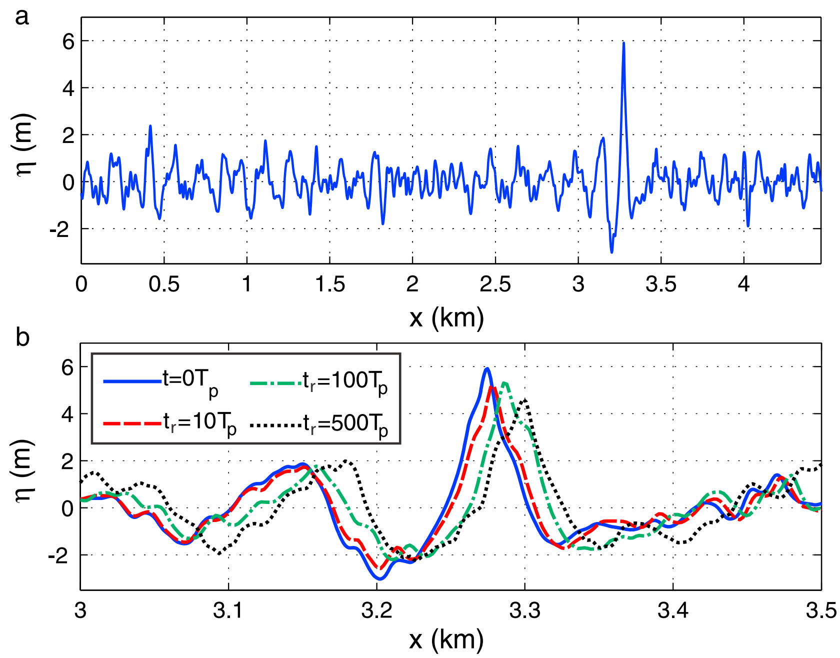


**Figure 1.** (a) A rogue wave with $H_{rs}$ = 2.78 in the sea state five. (b) Result of a $E_r$ = 10% error in the estimation of the initial state of the sea on the prediction of rogue wave. If sea state is known at $t_r$ = 10$T_p$ before the occurrence of the rogue wave, then a $E_r$ = 10% error has resulted in 13% error in the predictions (red dashed line). For $t_r$ = 100$T_p$ and 500$T_p$ in advance (green dash-dotted line and black dotted line), the same initial uncertainty results in, respectively, 30% and 28% errors. In the latter two cases the predicted wave is hardly a rogue wave by definition ($H_{ps}$ = 1.95 and 2.01). Simulation parameters are $N$ = 2048, $M$ = 4, and $T_p/\delta t$ = 128 for which presented results are converged.

multiple-scales approach) [cf. *Mei*, 1985] which is now backed by experimental proof [e.g. *Van Simaeys et al.*, 2002], however, reveals that after a threshold the process reverses and eventually the initial state is recovered (also cf. Fermi-Pasta-Ulam recurrence). Similar recursion is also known to exist and has validated experimentally, in the case of Peregrine solitons [*Peregrine*, 1983; *Shrira and Geogjaev*, 2009; *Akhmediev et al.*, 2011; *Chabchoub et al.*, 2013]. Therefore, the assumption of reversibility of water waves is not necessarily violated by the one-way initial exponential growth of perturbations. Computational results that follow consistently endorse this fact by showing excellent agreement in the reverse simulation.

## 3. Implementation, Results, and Discussion

For implementation of our three-stage procedure in a specific (given) sea state, we initialize our phase-resolved spectral scheme with amplitudes and frequencies given by the spectrum (4) and with random phases that have a uniform distribution. It is to be noted that when a nonlinear evolution equation is initialized by a linear initial condition, spurious high-frequency standing waves develop since the linear initial condition does not perfectly satisfy the nonlinear boundary conditions [cf. *Dommermuth*, 2000; *Carbone et al.*, 2013]. To minimize this effect, we gradually ramp up nonlinear terms in the governing evolution equations within a preinitial period chosen here to be $t_{\rm pre} = 5T_p$, where $T_p = 2\pi/\omega_p$ is the period of the peak frequency wave in the spectrum. Then, via running the direct computation for an initial time $t_0$ (typically $t_0 \sim O(100T_p)$), we search for those initial phases that lead to a rogue wave event within $0 < t_i < t_0$. We have performed $O(10^4)$ initial runs for each of the sea states four, five, and six (respectively moderate, rough, and very rough seas with $H_s$ = 1.875, 3.25, and 5.00 m and $T_p$ = 8.8, 9.7, and 12.4 s) and for each sea state have collected a database of $O(100)$ cases from those initial conditions that lead to rogue waves. Water surface of the sea state five at the time of occurrence of a rogue wave with $H_{rs} = H_r/H_s = 2.78$ ($H_r$ being the crest to trough height of the rogue wave) is shown in Figure 1a. Our computational experiments show that lower sea states do not develop rogue waves with very large values of $H_{rs}$. Specifically, in our database for the sea state four, Max($H_{rs}$)~2.6, whereas for sea states five and six we have recorded several rogue waves with $H_{rs}$ >2.6 (cf. Figure 3).

If an initial search is successful and a rogue wave is obtained at $t_i < t_0$, we then continue the evolution up to $t_f = t_i + t_r$ where $t_r = nT_p$ with $n \sim O(10^{2-3})$. As discussed above, a new simulation with initial conditions $(\eta_0, \phi_0) = (\eta(\mathbf{x}, t_f), -\phi(\mathbf{x}, t_f))$ will lead to a rogue wave exactly at $t = t_r$. In practice, this initial condition is an outcome of combined measurements and reconstruction procedures and is available with an inevitable *range of uncertainty*. To quantify the effect of uncertainty on the predictability of a rogue wave, we add Gaussian perturbations with a zero mean and standard deviation $E_r$ (in percent) to the amplitude and phases of components of the initial condition $(\eta_0, \phi_0)$. The perturbed state of the ocean, i.e., $\eta_{0p}$ and $\phi_{0p}$, is then used as the initial condition and its prediction at $t = t_r$ is compared with the actual rogue wave. In practice, if the perturbed initial condition predicts a *close-enough* approximation of the height, the location, and the time of occurrence of the rogue wave, most of the prediction objective is met. To take this fact into account, we search for the highest wave in the time span of $t_r - T_p < t < t_r + T_p$, and it is further checked that this highest wave is in the $\pm\lambda_p$ vicinity of the expected rogue wave ($\lambda_p$ is the wavelength associated with the peak period of the spectrum $T_p$). This highest predicted wave (with a trough to crest height of $H_p$) is what our predictor foresees at the vicinity of where the actual rogue wave will occur. We define $H_{ps} \equiv H_p / H_{s'}$ with $H_{s'}$ is the significant wave height as is calculated by the predictive simulation at the time of occurrence of $H_p$. Clearly, if $E_r = 0$, then $H_{ps} = H_{rs}$.

Effect of a $E_r = 10\%$ uncertainty in the initial condition on the prediction of the rogue wave of Figure 1a is shown in Figure 1b. The case of $t = 0$ corresponds to the original rouge wave. For $t_r > 0$, the longer the prediction time $t_r$, the more highlighted the effect of initial uncertainties becomes. Specifically, for $t_r = 10T_p$, $100T_p$, and $500T_p$, we have, respectively, $H_{ps} = 2.41$, 1.95, and 2.01 corresponding to 13%, 30%, and 28% error (note that uncertainty affects both numerator and denominator of the $H_{ps}$). In fact, predicted waves by the latter two cases ($t_r = 100T_p$ and $500T_p$) are hardly considered rogue waves by the definition. Note that Figure 1b shows the effect of *one* specific set of initial perturbations on the shape of the predicted rogue wave.

To highlight the significance of nonlinearities on the prediction and to also provide a convergence test for our scheme, we compare predictions initialized by $\eta_p, \phi_p$, for the rogue wave case presented in Figure 1a (sea state five), for $E_r = 0\%$, 5%, 10%, 15%, 20%, and 25%, and by taking different orders of nonlinearity (M) into account (Figure 2). To obtain a statistical average of the effect of uncertainty, each presented case (i.e., each marker in Figure 2) is the average result of 19 simulations each initiated with an independent set of random perturbations (the number 19 is chosen to meet error requirements as well as for an efficient use of computational resources at hand). All studied cases resulted in standard errors of less than 2%. In each panel of Figures 2a–2d we also consider four lead times of $t_r = nT_p$, $n = 1$, 10, 100, and 500.

Linear model ($M = 1$, Figure 2a) underpredicts the height of the rogue wave even for $E_r = 0\%$ and $n = 1$ (i.e., zero initial disturbance and within $t = T_p$). For $M = 2$, although for $n = 1$ predictions are acceptable, but for later times results are very much far from converged results of $M = 3$ and 4 (Figures 2c and 2d). A good convergence is observed for $M = 3$ (Figure 2c). The general behavior observed in Figures 2c and 2d is qualitatively the same for all cases we investigated.

To see if there is a quantitative trend in the predictability of oceanic rogue waves, we have performed extensive numerical experiments on $\sim O(100)$ rogue waves (documented in our database) for each of the sea states four, five, and six. Results for $t_r = 500T_p$ are shown in Figures 3a–3c, respectively, for sea states 4, 5, and 6. Each marker is again an average of 19 simulations with standard error of less than 2%.

Error in the prediction of the height of rogue waves, as suggested by Figure 3, is a function of height of the anticipated rogue wave, sea state, and, of course, degree of uncertainty. For all three sea states, the error in the prediction is higher for higher-amplitude rogue waves. The rate of increase in the error is larger for sea state five (Figure 3b) compared with sea state four (Figure 3a). This is due to the higher nonlinearity of the ambient waves in the sea state five (compared to sea state four) that further amplifies disturbances in the evolution equation. Prediction error, however, does not change further from sea state five to sea state six (Figure 3c) that implies an asymptotic saturation of how much the nonlinearity can contribute to the amplification of error over time.

We define predictability horizon for oceanic rogue waves at $H_{ps}/H_{rs} = 0.9$, i.e., when rogue wave height prediction can be made with 10% accuracy. This definition is based on the fact that 10% higher $H_{rs}$ corresponds to return period of an order of magnitude longer (in years) [cf. *Leonard-Williams and Saulter*, 2013]. Based on

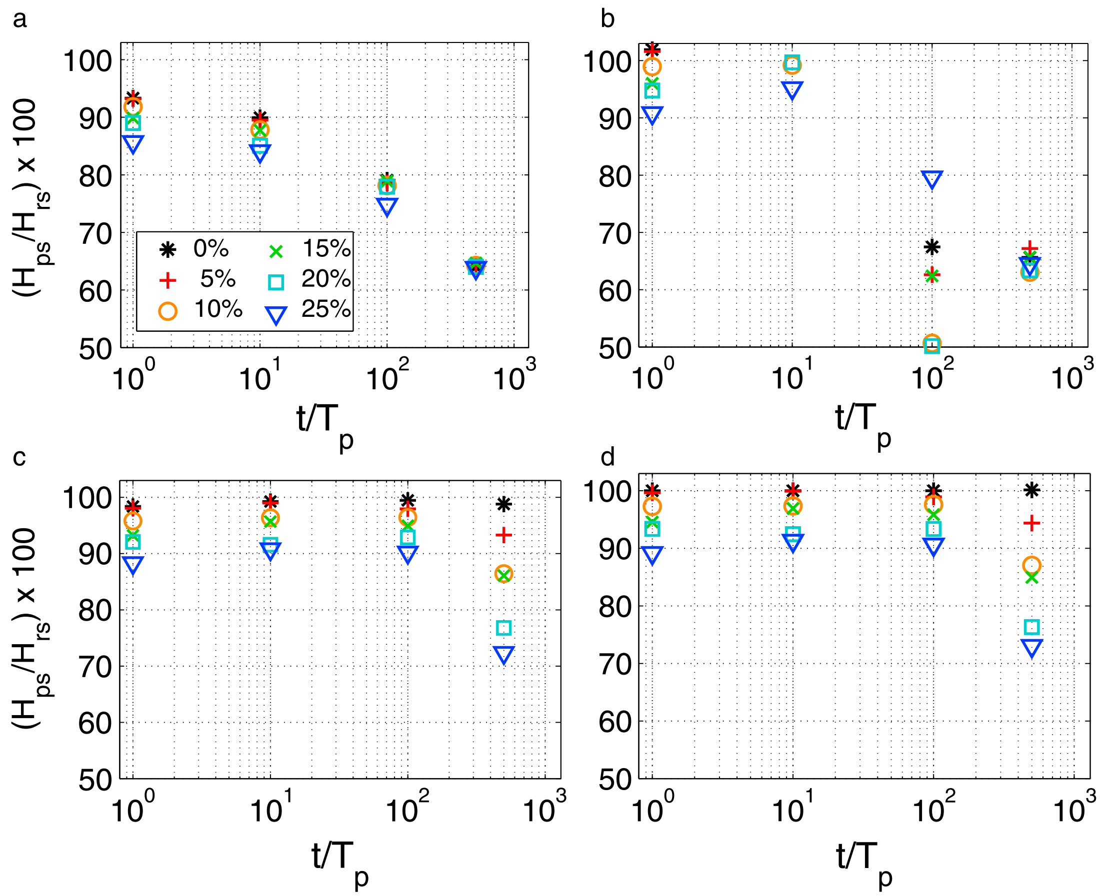


**Figure 2.** Effect of nonlinearities on the prediction of an oceanic rogue wave in a sea state five. (a–d) $M = 1$ (linear simulation), 2, 3, and 4 (second, third, and fourth order). Markers/colors correspond to different initial perturbations ($E_r$). For example, black star in Figure 2a shows that the linear model ($M = 1$), even with a zero initial uncertainty ($E_r = 0\%$), results in ∼5% error (i.e., $H_{ps}/H_{rs}$ ∼0.95) in the prediction of the height of the rogue wave which is only $t = T_p$ ahead. From Figures 2c and 2d it is seen that convergence is achieved for $M = 3$. Standard error of all data points is less than 2%. Other simulation parameters are the same as in Figure 1.

this definition, a rogue wave in a sea state four is predictable $500T_p$ ahead of occurrence if the uncertainty in the sea state measurement at the current time is less than 20%. To achieve this level of predictability in a sea states five and six, the uncertainty has to be less than ∼5%.

In practice, many offshore structures are designed today for the extreme waves of return period 10,000 years. It is crucial for these structures to know if a rogue wave with a height greater than the design value is to occur at their location. If this knowledge is in hand, several precautionary procedures can be carried out to minimize the damage and the potential loss of life (such procedures include, for instance, shutdown or relocation). Therefore, a critical question is if a reliable prediction can be made. Norwegian offshore standard NORSOK [*NORSOK-N-003*, 2007] suggests that in the absence of more detailed information, an extreme wave of $H_{10,000}/H_s = 2.375$ has an annual probability of occurrence of less than $10^{-4}$. Therefore, in Figures 3a–3d, waves with $H_{rs} > 2.375$ are larger than $H_{10,000}$. Markers that fall inside the gray area show rogue waves that are in fact larger than $H_{10,000}$ (i.e., dangerous to the structure) but are predicted to be smaller than $H_{10,000}$ (i.e., falsely predicted to be safe waves). Figures 3b and 3c show that with more than 10% uncertainty in the estimation of wave components of ocean states five and six, rogue waves higher than $H_{10,000}$ cannot be predicted sooner than $500T_p$ ahead of time.

We would like to remark that the term "horizon of predictability" is used extensively, though not exclusively [cf. *López et al.*, 2011], in the context of chaotic dynamical systems. It is to be noted that although water surface may undergo chaotic motion in cases [e.g *Alam et al.*, 2009a; *Hadjihosseini et al.*, 2014], the use of the term *predictability horizon* here is not based on such behavior but merely significant temporal amplification of noise by nonlinearities. Whether water surface undergoes chaotic behavior, and if so under what condition(s), is an interesting subject of research but requires a separate investigation.

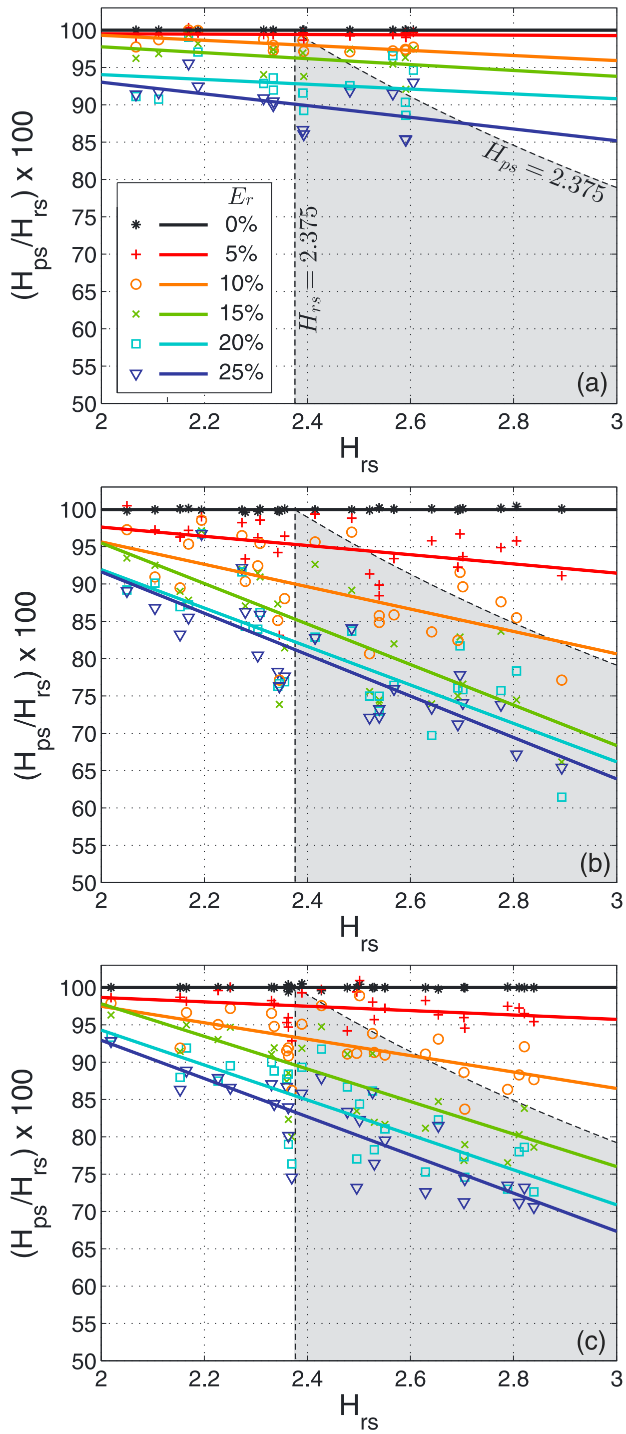


**Figure 3.** Predictability of oceanic rogue waves as a function of rogue wave's height and uncertainty in the initial condition ($t_r = 500T_p$) for, sea states (a) four, (b) five and (c) six. For nonzero perturbations, error in prediction is larger if amplitude of the rogue wave is larger and if the sea state is higher (cf. Figures 3a and 3b) but reaches an asymptotic saturation at the sea state six (cf. Figures 3b and 3c). Shaded areas are particularly important areas for practical applications because actual rogue waves with $H_{rs} > 2.375$ (on the right of the vertical dashed line) are stronger than 10,000 year design standards, but for markers that fall inside the shaded area, the prediction incorrectly underestimates the amplitude to be safe. Other simulation parameters are the same as in Figure 1.

The spectrum considered in this letter is (relatively) broad. It is known, however, that narrower spectra are more amenable to instabilities [e.g., *Waseda et al.*, 2011], and therefore, it is expected that the predictability is weaker for a narrower spectrum sea state.

Propagation of waves in the real ocean is also affected by the winds blowing over the surface of the ocean, turbulent boundary layers, oceanic currents, and dissipation due to wave breaking. These so-called *external action and losses* are known to influence the oceanic spectral evolution [e.g., *Komen et al.*, 1994; *Young*, 1999; *Tulin et al.*, 1999; *Janssen*, 2004; *Babanin*, 2011] and hence the probability and predictability of oceanic rogue waves [e.g., *Xiao et al.*, 2013; *Onorato et al.*, 2009]. Despite the great volume of research over past decades, our understanding of these effects is very limited. Full consideration of these external action and losses requires a direct Computational Fluid Dynamics (CFD) analysis that for the scales of our interest is beyond existing computational capabilities.

The energy loss due to wave breaking in inviscid potential flow models is usually considered by approximate damping terms introduced in the governing equations. For example, nonlinear Schrödinger equation can turn into a dissipative equation by adding a linear term in the envelope amplitude with the dissipation coefficient in front. The added term, however, does not reflect any specific physics but just dissipates energy due to breaking and/or bottom friction [e.g. *Voronovich et al.*, 2008]. Estimation of coefficient comes from matching the numerical evolution of a wave group with the experimental studies [e.g., *Rapp and Melville*, 1990; *Gemmrich and Farmer*, 1999; *Wu et al.*, 2004; *Kharif et al.*, 2009; *Xiao et al.*, 2013; *Tian et al.*, 2012; *Tian and Choi*, 2013]. Wave breaking is expected to lower the chance of occurrence of oceanic rogue waves and hence positively contribute to the predictability horizon. Wind effects are considered similarly by implementing a pressure distribution over the surface whose phases and amplitudes are determined semiempirically [e.g. *Montalvo et al.*, 2013; *Tian and Choi*, 2013; *Kalmikov*, 2010][*Montalvo et al.*, 2013; *Tian and Choi*, 2013; *Kalmikov*, 2010]. Effect of wind on the formation of rogue wave is a matter of dispute. Some studies show that blowing winds can stabilize Benjamin-Feir instability, hence reducing the probability of rogue wave formation [e.g. *Bliven et al.*, 1986; *Li et al.*, 1987], while others conclude that wind cannot inhibit the sideband growth [e.g. *Waseda et al.*, 1999; *Novgorod et al.*, 2008].

## 4. Concluding Remarks

Here we defined a quantitative predictability horizon and calculated a statistically converged predictability time scale for oceanic rogue waves in two-dimensional unidirectional broadband seas. This horizon is shorter in higher sea states and if the amplitude of the actual anticipated rogue wave is higher. Nonlinearity and nonlinear interactions are the major players behind the amplification of the initial uncertainty and affect the prediction to the extent that all major features of an upcoming rogue wave may be completely lost.

Sensitivity of predictions of analytical model equations such as nonlinear Schrödinger [*Toffoli et al.*, 2011; *Akhmediev et al.*, 2009a, 2009b, 2010; *Chabchoub et al.*, 2011] and its analytical recursive solutions (e.g., Peregrine soliton [*Peregrine*, 1983; *Shrira and Geogjaev*, 2009; *Kibler et al.*, 2010] or Akhmediev breather [*Mahnke and Mitschke*, 2012; *Vishnu Priya et al.*, 2013]), and specific growth mechanisms such as Benjamin-Feir instability [*Benjamin and Feir*, 1967; *Xiao et al.*, 2013; *Lake et al.*, 1977] (particularly in broadband) to the initial perturbations, may provide analytical predictability horizon and merits thorough investigation in prospective studies.

Another important and immediate follow-up question is the predictability horizon in three dimension and how presented results here will be affected. In three dimension, spreading angle (directional bandwidth) enters the initial condition as a new parameter. An increase in the spreading angle is known to decrease the probability of occurrence of rogue waves [*Onorato et al.*, 2002; *Gramstad and Trulsen*, 2007; *Waseda et al.*, 2009; *Onorato et al.*, 2009]. Specifically, in short-crested seas (i.e., crest lengths be of the same order as the wavelength) statistics of freak waves remains more or less Gaussian. In this case a second-order stokes theory can very well obtain the exceedance probability of wave crests [*Tayfun*, 1980]. However, for long-crested waves (i.e., crest lengths an order of magnitude larger than the wavelength) statistics of freak waves deviates considerably from Gaussian distribution, second-order theory fails to predict the probability, and probability of occurrence of rogue waves correlates with the Benjamin-Feir index which is less determinant in short-crested seas [*Mori et al.*, 2011]. Therefore, while long-crested assumption used here is a more conservative assumption in terms of probability of occurrence, care must be taken in the extension of presented sensitivity results to the short-crested seas.

Rogue waves may appear not only in surface gravity wave systems but also in optical systems [*Solli et al.*, 2007; *Dudley et al.*, 2010], capillary waves [*Shats et al.*, 2010], and in plasma physics [*Moslem et al.*, 2011], where techniques developed here and results obtained may have similar implications.

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