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A FLEXIBLE SEAFLOOR CARPET FOR HIGH-PERFORMANCE WAVE ENERGY EXTRACTION

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ABSTRACT

Similar to the mechanism by which a visco-elastic mud damps the energy of overpassing surface waves, if the near-shore seafloor is carpeted by an elastic thin material attached to generators (i.e. dampers) a high fraction of surface wave energy can be absorbed. Here we present analytical modeling of the flexible carpet wave energy converter and show that a high efficiency is achievable. Expressions for optimal damping and stiffness coefficients are derived and different modes of oscillations are discussed. The presented wave energy conversion scheme is completely under the water surface hence imposes minimal danger to boats and the sea life (i.e. no mammal entanglement). The carpet is survivable against high momentum of storm surges and in fact can perform well under very energetic (e.g. stormy) sea conditions, when most existing wave energy devices are needed to shelter themselves by going into an idle mode.

I am honored to be a colleague of Prof. Ronald Yeung at the University of California, Berkeley. He is a world renowned scientist of ship hydrodynamics with several valuable and key contributions to the field. This manuscript on a new ocean wave energy extraction scheme is due to Ron's recent interest in the field of ocean renewable energy. I am looking forward to years of working closely with him. Thank you Ron.

INTRODUCTION

Gade (1) reports a place in the gulf of Mexico known to locals as *mud hole* where due to the accretion of mud banks has turned into, for the local fishermen, a safe haven against strong waves during storms. Within the mud hole the interaction of surface waves with the mud is very strong such that waves completely damped out within a few wavelengths (2). Observations of strong wave damping due to the coupling with the bottom mud is not limited to the gulf, but almost anywhere with a muddy seafloor (e.g. 3; 4; 5).

If mud can take a substantial energy out of incident surface gravity waves, an artificial carpet deployed on the seabed that responds to the action of the overpassing waves in the same way as the response of a mud-layer must be able to extract the same amount of energy. Analysis of performance of this synthetic seabed-carpet wave energy conversion technique is the subject of this article.

The complicated nature of the seafloor mud and the wide range of mechanical/material properties which may be location and even time dependent, has aroused a great deal of research on this subject in the past. For understanding wave-mud interactions several models have been incorporated including, but not limited to, Newtonian (6), non-Newtonian (7; 8), viscoelastic (9; 10), porous (11), poro-elastic (12), and bottom friction (13). The correct model in general is yet a matter of dispute (14; 15), however, under the periodic forcing a viscoelastic model has been shown to be a very good approximation and is now widely used (16; 17). While the general idea presented here can incorporate any mud model, for specificity, we focus our attention on a linear viscoelastic model.

In this paper we consider a seabed-carpet composed of carpet mass attached to sets of vertically acting linear springs and generators, with the generator's action modeled to be linearly proportional to the vertical speed. We show that the coupled governing equation of waves/carpet system admits two modes

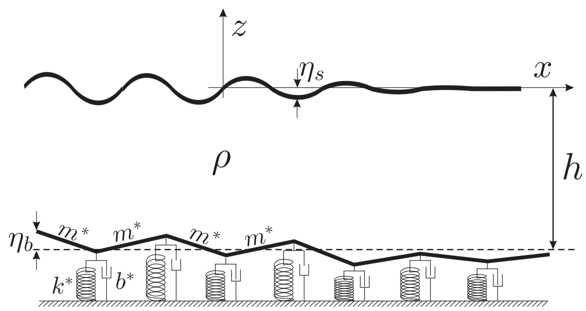

Figure 1. Schematic of the configuration considered in this paper. A visco-elastic carpet is mounted on the seafloor that extracts energy from overpassing surface waves. The carpet (with distributed mass m^*) is restricted by distributed springiness and damping coefficients of k^*, b^* respectively.

of propagating wave solution (§1): a surface mode wave whose frequency and wavenumber in the limit of deep water tend to the frequency and wavenumber of free propagating surface gravity waves, and a bottom-mode wave whose frequency is much higher (for a relatively stiff bottom) or much lower (for a relatively flexible bottom) than the surface wave of the same wavenumber. For a surface-mode the higher the wavelength, the higher the energy extraction by the bottom. This is in agreement with most observations of long waves damped by the bottom mud (7). For a bottom-mode wave, however, the energy extraction increases as the wavelength decreases (§2,3). Aside from attractiveness of this feature of a bottom-mode wave for the wave energy community, it may also provide an explanation for the recent observation (yet unexplained) of strong short wave damping by (5)(see also 18).

Since the idea presented here is more efficient in shallow waters, and since due to shoaling the effect of nonlinearity increases in shallower depths, we further formulate the weak nonlinear problem for long waves (§3). Governing equations up to second order are presented and discusses are provided.

The presented wave energy conversion device is completely under the water surface hence imposes minimal danger to boats and the sea life (i.e. no mammal entanglement). The carpet is survivable against high momentum of storm surges and in fact can perform even better under very energetic (e.g. stormy) sea conditions, when most existing wave energy devices are needed to shelter themselves by going into an idle mode. The proposed idea and its variations may also be used to create localized safe havens for fishermen and sailors in open seas, or if implemented in large scales to protect shores and harbors against strong storm waves.

1 Governing Equation

We consider the irrotational motion of a homogeneous inviscid incompressible fluid with a free surface over a flexible carpet placed at the mean bottom $z = -h$. Linearized governing equations ignoring surface tension is

$$\nabla^2\phi = 0 \tag{1.1a}$$
$$\phi_{tt} + g\phi_z = 0 \quad @\ z = 0, \tag{1.1b}$$
$$\phi_z - \eta_{b,t} = 0 \quad @\ z = -h, \tag{1.1c}$$
$$m^*\eta_{b,tt} + b^*\eta_{b,t} + k^*\eta_b + P_b = 0 \quad @\ z = -h, \tag{1.1d}$$
$$\phi_t + g\eta_b + \frac{P_b}{\rho} = 0 \quad @\ z = -h. \tag{1.1e}$$

where m^*, b^* and k^* are respectively mass, viscous damping and stiffness coefficient per unit length. The combination of the last three boundary conditions (1.1c)-(1.1e) give

$$m^*\phi_{,ztt} + b^*\phi_{,zt} + (k^* - \rho g)\phi_{,z} - \rho\phi_{,tt} = 0 \quad @\ z = -h \tag{1.2}$$

General solution to the Laplace's equation (1.1a) is

$$\phi = \left(Ae^{kz} + Be^{-kz}\right)e^{i(kx-\omega t)} \tag{1.3}$$

where upon substitution into the free surface boundary condition (1.1b) yields

$$B = A\frac{gk - \omega^2}{gk + \omega^2} \tag{1.4}$$

and from (1.2) we obtain

$$(\mu + \mathcal{R}\tanh\mu)\Omega^4 + 2i\mu\zeta\Omega_b\Omega^3 - \mu(\Omega_b^2 + \mu\tanh\mu)\Omega^2 - 2i\mu^2\zeta\Omega_b\Omega\tanh\mu + \mu^2(\Omega_b^2 - \mathcal{R})\tanh\mu = 0, \tag{1.5}$$

which is the dispersion relation in terms of dimensionless variables

$$\Omega = \omega\sqrt{h/g}, \quad \mathcal{R} = \frac{\rho h}{m^*}, \quad \zeta = \frac{b^*}{2\sqrt{k^* m^*}},$$
$$\mu = kh, \quad \Omega_b = \sqrt{\frac{k^*}{m^*}}\sqrt{h/g}.$$

where Ω is the dimensionless frequency, $\mathcal{R}$ is the ratio of the mass of fluid above to the mass of the carpet (for a given area), ζ is dimensionless damping ratio, μ is shallowness and Ω_b is the dimensionless bottom natural frequency (in the absence of fluid on top). Note that limiting cases of $k^* \to \infty$ and $m^* \to \infty$ are equivalent of a rigid bottom and (1.5) readily reduces to the flat bottom dispersion relation.

In the limit of deep water ($\mu \gg 1$) equation (1.5) is further simplified to

$$(\Omega^2 - \mu)\left[(\mu + \mathcal{R})\Omega^2 + 2i\mu\zeta\Omega_b\Omega + \mu(\mathcal{R} - \Omega_b^2)\right] = 0. \quad (1.6)$$

The first parenthesis shows the asymptotic convergence of the surface-mode to the deep water wave dispersion relation while its damping goes to zero.

The first parenthesis is simply the dispersion relation of deep water waves. Clearly bottom damping has no effect on waves in this mode. The second parenthesis is the dispersion relation of what we call the *carpet mode*. We will show later that in this mode, converse to the surface mode, effect of damping *increases* as the water depth increases. For stable propagating waves, i.e. to have non-growing results, we need to always have $\Im(\Omega) < 0$ (c.f. (1.3)). For this to satisfy in (1.6) it can be shown that it is necessary to have

$$\Omega_b^2 < \mathcal{R}. \quad (1.7)$$

In physical space this requirement is equivalent to $k^* > \rho g$ and means the restoring force acting on any perturbation on the carpet has to be higher than the weight added to the carpet due to that perturbation. Under condition (1.7) and in the absence of the dissipation (i.e. $\zeta = 0$) equation (1.6) has two pairs of real solutions (c.f. fig. 2a): the surface wave mode for which surface amplitude is higher than carpet amplitude but surface and bottom undulations are in phase, and, the carpet mode for which bottom amplitude is higher but surface and bottom undulations have π radian phase difference (fig. 2b). As $\mu \to \infty$ surface mode frequency Ω_s increases proportional to $\Omega_{s\infty} \propto \sqrt{\mu}$ while carpet mode frequency Ω_c asymptotically tends to $\Omega_{c\infty} \propto (\Omega_b^2 - R)$. Also from (1.5) in the limit of $\zeta = 0, \mu = 0$ it can be shown that $\Omega_{s0} = \Omega_b/\sqrt{1+R}$, therefore no wave with the frequency $(\Omega_b^2 - R) < \Omega < \Omega_b/\sqrt{1+R}$ can exist. The band-gap is shaded with gray in fig. 2a.

Now let's consider the case where $\zeta \neq 0$. In this case and if bottom-mode roots of (1.6) are sought, it can be further shown that a bifurcation in behavior occurs at a critical damping ratio

$$\zeta_{cr} = \sqrt{\frac{(\Omega_b^2 - \mathcal{R})(\mu + \mathcal{R})}{\mu\Omega_b^2}}. \quad (1.8)$$

For $\zeta > \zeta_{cr}$ and for any given μ we obtain that $\Omega_c = ix_1, ix_2$ where $x_1, x_2 \in \mathbb{R}^-, x_1 \neq x_2$. If $\zeta < \zeta_{cr}$ then $\Omega_c = ix_3 \pm x_4$ where $x_3, x_4 \in \mathbb{R}^-$. Clearly at $\zeta = \zeta_{cr}$ the oscillatory motion of the bottom-mode diminishes and waves decay exponentially to zero.

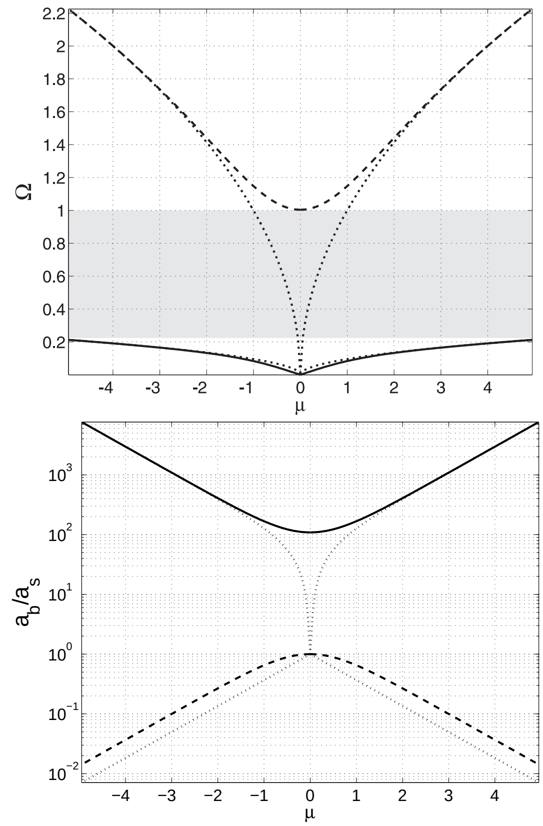

Figure 2. (a) Roots of the dispersion relation in the absence of damping (ζ =0). Parameters are $\mathcal{R} = 1000$ and Ω_b =32. Plotted are roots of (1.5) (——), and (1.6) (· · ·). Shaded region is the band-gap for which corresponding frequencies do not exist. (b) Ratio of bottom to surface amplitudes a_b/a_s for which the same notation as in (a) is used.

Real and imaginary part of the solution Ω of (1.5) for a fixed $\mathcal{R}$=1000, Ω_b=0.32 and ζ=1.7,3.4 are shown in figure 3, along with the bottom to surface amplitude ratios and deep water asymptotes.

The real part of frequency of surface mode waves, $\Re(\Omega_s)$, experience minimal change compared to the undamped case. The imaginary part of a surface mode wave, $\Im(\Omega_s)$, starts from a finite negative value (corresponds to finite decay rate) and asymptotically decreases to zero as waves get shorter. This is expected and in agreement with existing theories and observations of stronger long wave (compared to short wave) damping. The behavior of the bottom mode waves are however more complex. As explained earlier roots of the bottom mode bracket of (1.5) show a bifurcation at a critical (dimensionless) depth.

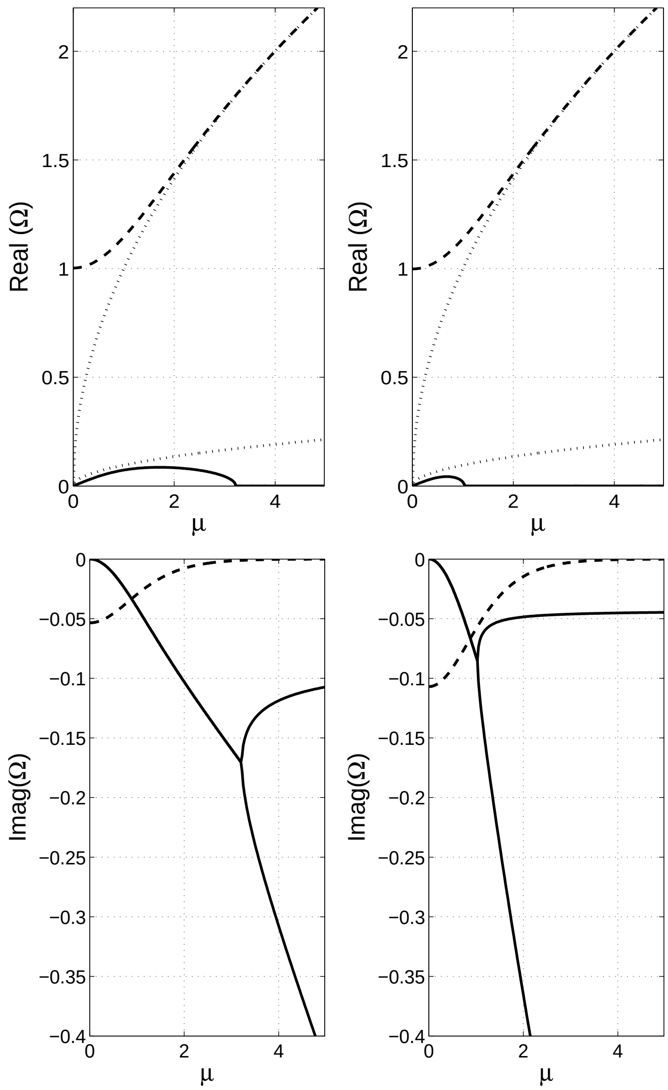

Figure 3. see fig. 4 for the caption

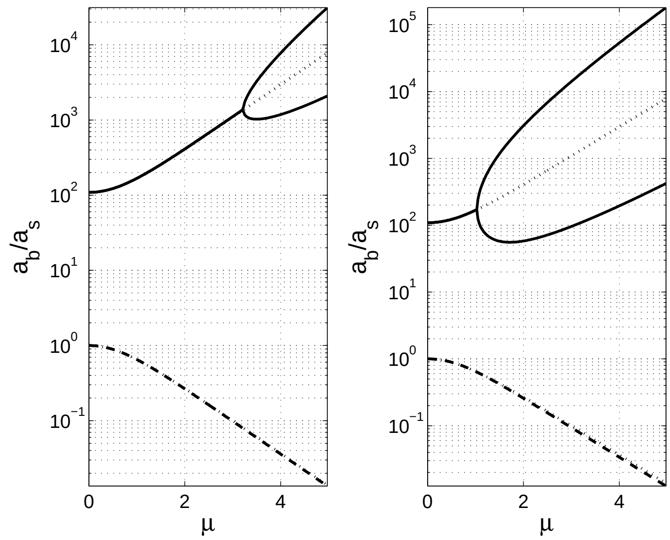

Figure 4. Real and imaginary branches of the solution $\Omega(\mu)$ to the dispersion relation (1.5) for $\mathcal{R} = 1000$, $\Omega_b = 0.32$ and $\zeta = 1.7$ (left column) and $\zeta = 3.4$ (right column). Also plotted is the ratio of the bottom amplitude to the surface amplitude. Dashed curves are branches associated with the surface mode and solid curves are associated with the bottom mode. Note the bifurcation at μ_{cr}=3.62 and μ_{cr}=1.23 (c.f.(1.8)). Branches of deep-water asymptotes (1.6) are also plotted for comparison $(\cdot\ \cdot\ \cdot)$.

2 Approximate damping rate

we consider wave in a homogeneous fluid bounded by periodic side-boundary conditions, free surface and a visco-elastic bed. In two dimension, the governing equation for the bottom is

$$m^*\ddot{\eta}_b + b^*\dot{\eta}_b + k^*\eta_b = p(t) \tag{2.9}$$

where m^*, b^* and k^* are mass, damping coefficient and stiffness coefficient of the bottom per unit length per unit depth. $p(t)$ is the pressure at the mean bottom $z = -h$.

If pressure is periodic in time, say $p(t) = \tilde{p}_0 \cos(\omega t)$, the solution to this ordinary differential equation is well known

$$\eta_b = \frac{\tilde{p}_0}{[(k^* - m^*\omega^2)^2 + {b^*}^2\omega^2]^{\frac{1}{2}}} \cos(\omega t - \phi) \tag{2.10}$$

where the phase shift is given by

$$\phi = \tan^{-1}\left(\frac{b^*\omega}{k^* - m^*\omega^2}\right) \tag{2.11}$$

The power loss per unit horizontal length due to the damping of the bed is

$$P_{ow}(x,t) = b^*\ \dot{\eta}_b^2 = p(t)\ \dot{\eta}_b \tag{2.12}$$

and the time average power loss is given by

$$\bar{P}_{ow} = \frac{1}{2}\frac{b\tilde{p}_0{}^2\omega^2}{[(k - m\omega^2)^2 + (b\omega)^2]} \tag{2.13}$$

For a monochromatic progressive wave, this is the power loss per unit length. For standing wave, the power loss is half of this value due to another averagin over the wave length that introduce a factor of one-half.

The energy contained in one wave-length of a progressive wave

$$\eta = a\sin(kx-\omega t) \tag{2.14}$$
$$\phi = -\frac{ag}{\omega}\frac{\cosh k(z+h)}{\cosh kh}\cos(kx-\omega t) \tag{2.15}$$

is

$$E_\lambda = \frac{1}{2}\rho g a^2 \lambda \tag{2.16}$$

or $E_w = 1/2\rho g a^2$ per unit length (see for example 19, section 2.2). Note that the energy of standing wave is half of propagatin wave with the same amplitude and wave number.

Pressure at the mean bottom is

$$p(x,t) = -\rho[\phi_{,t} + gz] \quad @z=-h \tag{2.17}$$
$$= \frac{\rho g a}{\cosh kh}\sin(kx-\omega t) + \rho g h \tag{2.18}$$

The first terms is the dynamic pressure due to the presense of the wave hence decreases as depth increases. At the limit when $kh \ll 1$ the hydrostatic pressure of shallow water wave is recovered. The second term is just the hydrostatic pressure due to the water column and only changes the set point of our mass-spring-damper system and therefore does not play any role in the damping process. To find the rate of amplitude decay we write

$$\frac{dE_w}{dt} = \bar{P} \quad \Rightarrow \frac{da}{dt} = -C\,a \quad \Rightarrow a = a_0 e^{-Ct} \tag{2.19}$$
$$C = \frac{1}{2\cosh^2 kh}\cdot\frac{\rho g b^* \omega^2}{[(k^*-m^*\omega^2)^2 + (b^*\omega)^2]} \tag{2.20}$$
$$= \frac{1}{2\cosh^2 kh}\cdot\frac{2\zeta\kappa\tilde{\Omega}}{[(1-\tilde{\Omega}^2)^2 + (2\zeta\tilde{\Omega})^2]}\cdot\omega \tag{2.21}$$

where $\kappa = \rho g/k^*$

3 Nonlinear Shallow Water

The idea presented here is more likely to be employed in a shallow water regime. For this purpose in this section we derive weakly nonlinear shallow water equations governing wave propagation over a visco-elastic bottom carpet. Full nonlinear governing equations of waves over time-dependent bottom is given

$$\nabla^2\phi = 0 \qquad -h(x,t) < z < \eta_s \tag{3.22a}$$
$$\eta_{s,t} + \eta_{s,x}\phi_{,x} = \phi_{,z} \qquad z = \eta_s \tag{3.22b}$$
$$\phi_{,t} + \tfrac{1}{2}(\nabla\phi)^2 + g\eta_s = 0 \qquad z=\eta_s \tag{3.22c}$$
$$-h_{,t} - h_{,x}\phi_{,x} = \phi_{,z} \qquad z = -h(x,t) \tag{3.22d}$$

where $-h = -h_0 + \eta_b$ and h_0 is the average bottom depth. The following scales are introduced to dimensionless the governing equations

$$x' = kx,\quad z' = \frac{z}{h_0},\quad t' = k\sqrt{gh_0}t,\quad \eta_s' = \frac{\eta_s}{a},\quad \eta_b' = \frac{\eta_b}{a},$$
$$\phi' = \frac{kh_0}{a\sqrt{gh_0}}\phi,\quad h' = \frac{h}{h_0},\quad P' = \frac{P}{\rho g h_0} \tag{3.23}$$

where k, a and g are respectively the characteristic wave-number, characteristic amplitude and the acceleration gravity and P is the pressure. Upon substitution into the governing equation, after dropping prims we get

$$\mu^2\phi_{,xx} + \phi_{,zz} = 0 \qquad -h < z < \varepsilon\eta_s \tag{3.24a}$$
$$\mu^2(\eta_{s,t} + \varepsilon\eta_{s,x}\phi_{,x}) = \phi_{,z} \qquad z = \varepsilon\eta_s \tag{3.24b}$$
$$\mu^2(\phi_{,t} + \eta_s) + \tfrac{1}{2}\varepsilon(\mu^2\phi_{,x}^2 + \phi_{,z}^2) = 0 \qquad z = \varepsilon\eta_s \tag{3.24c}$$
$$-\mu^2(h_{,t} + \varepsilon h_{,x}\phi_{,x}) = \phi_{,z} \qquad z = -h \tag{3.24d}$$

where

$$\mu \equiv kh_0 \ll 1, \quad \varepsilon \equiv \frac{a}{h_0} \ll 1 \tag{3.25}$$

are indicators of the shallowness and the nonlinearity respectively. The ratio of nonlinearity to the shallowness

$$U_r = \frac{\varepsilon}{\mu^2} \tag{3.26}$$

is called the Ursell's number. We assume a solution in the form

$$\phi = \sum_0^\infty [z + h(x,t)]^n \phi_n(x,y) \tag{3.27}$$

From Laplace's equation i.e., equation (3.24a) we have

$$\phi_{n+2} = -\mu^2\frac{\phi_{n,xx} + 2(n+1)h_{,x}\phi_{n+1,x} + (n+1)h_{,xx}\phi_{n+1}}{(n+1)(n+2)[1+\mu^2 h_{,x}^2]} \tag{3.28}$$

and the boundary condition equation (3.24d) gives

$$\phi_1 = -\frac{1}{U_r}\,\frac{h_{,t}+\varepsilon h_{,x}\phi_{0,x}}{1+\mu^2 h_{,x}^2} \tag{3.29}$$

Assuming $U_r = O(1)$ and $h = 1-\mu^\alpha \eta_b(x,t)$ where $\alpha \geq 1$, we conclude $O(h_{,x} = O(h_{,x}) = O(\mu^\alpha) \ll 1$ after some algebra we get to the final equations correct to the order of $O(\mu^2) = O(\varepsilon)$:

$$\mathcal{H}_{,t} + \varepsilon(\mathcal{H}\bar{u})_{,x} = 0 \tag{3.30}$$

$$\bar{u}_{,t} + \varepsilon \bar{u}\bar{u}_{,x} + \eta_{s,x} - \frac{\mu^2}{3}\bar{u}_{,xxt} + f(x,t) = 0 \tag{3.31}$$

$$P = (\varepsilon\eta_s - z) + O(\varepsilon\mu^\alpha) \tag{3.32}$$

where

$$\mathcal{H} = h + \varepsilon\eta_s, \qquad \bar{u} = \frac{1}{\mathcal{H}}\int_{-h}^{\varepsilon\eta_s} \phi_{,x} dz \tag{3.33}$$

are the total depth from the free surface and the depth average velocity, and

$$\begin{aligned} f(x,t) &= \frac{1}{U_r}\left[h_{,x}h_{,tt} + \frac{1}{2}\mathcal{H}_{,t}h_{,xt} - \frac{1}{2}\mathcal{H}\, h_{,xtt} - \mathcal{H}_{,x}h_{,tt}\right] \\ &= \frac{1}{2U_r}\left[\mu^\alpha \eta_{b,xtt} + \mu^{2\alpha}(\eta_{b,t}\eta_{b,xt} - \eta_b\eta_{b,xtt})\right] \end{aligned} \tag{3.34}$$

when the bottom is not a function of time, equations (3.30) and (3.31) reduce to equations (12.1.47) and (12.1.48) of (20) with the assumption of small amplitude topography. Close to the bottom, the error in the pressure term drops to $O(\varepsilon\mu^{2\alpha})$.

Assuming $\alpha = 2$, i.e., the bottom variation is as big as the surface perturbations, we can combine equations (3.30) and (3.31) to get

$$\begin{aligned} \eta_{s,tt} - \eta_{s,xx} - \frac{1}{U_r}\eta_{b,tt} = \varepsilon\{(uu_x)_{,x} - (\eta_s u)_{,xt}\} + \\ \mu^2\left\{(\eta_b u)_{,xt} - \frac{1}{3}u_{,xxxt} + \frac{1}{2U_r}\eta_{b,xxtt}\right\} \end{aligned} \tag{3.35}$$

the bottom governing equation in the dimensional space is

$$m^*\eta_{b,tt} + b^*\eta_{b,t} + k^*\eta_b = -P = \rho g(\eta_s - \eta_b) \tag{3.36}$$

where m^*, b^* and k^* are respectively mass, damping coefficient and stiffness coefficient per unit area in a three dimensional problem. Note that we assume η_b is measured from the equilibrium position of the bottom where spring resists the pressure of still water $\rho g h_0$. In dimensionless form

$$\alpha_1\eta_{b,tt} + \alpha_2\eta_{b,t} + (\alpha_3 - 1)\eta_b = -\eta_s \tag{3.37}$$

where

$$\alpha_1 = \mu\frac{km^*}{\rho}, \quad \alpha_2 = \mu\frac{b^*}{\rho\sqrt{gh}}, \quad \alpha_3 = \frac{k^*}{\rho g}. \tag{3.38}$$

Now we introduce two space variables x and $X = \mu^2 x$ and expand η_s, η_b and u in power series as follows

$$\eta_s(x,X;t) = \eta_{s0} + \mu^2\eta_{s1} + O(\mu^4) \tag{3.39}$$

$$\eta_b(x,X;t) = \eta_{b0} + \mu^2\eta_{b1} + O(\mu^4) \tag{3.40}$$

$$u(x,X;t) = u_0 + \mu^2 u_1 + O(\mu^4) \tag{3.41}$$

upon substitution into equation 3.35, the perturbation equations are obtained

$$\eta_{s0,tt} - \eta_{s0,xx} - \frac{1}{U_r}\eta_{b0,tt} = 0 \tag{3.42}$$

$$\begin{aligned} \eta_{s1,tt} - \eta_{1,xx} - \frac{1}{U_r}\eta_{b1,tt} &= \varepsilon\{(u_0u_{0,x})_{,x} - (\eta_{s0}u_0)_{,xt}\} \\ &+ \mu^2\Big\{ 2\eta_{s0,xX} + (\eta_{b0}u)_{,xt} \\ &- \frac{1}{3}u_{0,xxxt} + \frac{1}{2U_r}\eta_{b0,xxtt}\Big\} \end{aligned} \tag{3.43}$$

Using equation (3.37) zeroth order equation (3.42) can be written in terms of a single variable η_{b0}

$$\begin{aligned} \alpha_1(\eta_{b0,tttt} - \eta_{b0,ttxx}) + \alpha_2(\eta_{b0,tt} - \eta_{b0,xx})_{,t} + \\ (\alpha_3 - 1 + U_r^{-1})\eta_{b0,tt} - (\alpha_3 - 1)\eta_{b0,xx} = 0. \end{aligned} \tag{3.44}$$

If the damping coefficient is zero, the solution to the linear problem is given by

$$\eta_s = \eta_{s0}e^{i(kx-\omega t)} \tag{3.45}$$

with

$$\eta_b = \eta_{b0}e^{i(kx-\omega t)} \tag{3.46}$$

$$u = u_0 e^{i(kx-\omega t)} \tag{3.47}$$

where in dimensional form

$$\eta_{b0} = \frac{\rho g}{m^*\omega^2 - (k^* - \rho g)}\eta_{s0} = \left(1 - \frac{ghk^2}{\omega^2}\right)\eta_{s0}, \quad (3.48)$$

$$u_0 = \frac{gk}{\omega} \quad (3.49)$$

and has a dispersion relation in the form

$$\omega^4 - \omega^2\left(ghk^2 + \frac{k^*}{m^*}\right) + ghk^2\frac{k^* - \rho g}{m^*} = 0. \quad (3.50)$$

In dimensionless format we get

$$\eta_{s0} = \eta^0_{s0}e^{i(kx-\omega t)}, \quad \eta^0_{b0} = \left(1 - \frac{1}{c^2}\right)\eta^0_{s0}, \quad (3.51)$$

$$u^0_0 = \frac{1}{c}\eta^0_{s0} \quad (3.52)$$

where $c = \omega/(k\sqrt{gh})$ is the dimensionless wave speed and $a\sqrt{gh}/h$ is used to nondimensionlize the velocity u. Therefore

$$\eta_{s0} = \zeta(\sigma), \quad \eta_{b0} = \left(1 - \frac{1}{c^2}\right)\zeta(\sigma), \quad u_0 = \frac{1}{c}\zeta(\sigma) \quad (3.53)$$

where $\sigma = x - ct$.

Since the dispersion relation (3.50) is a nonlinear equation, in general, $(2k, 2\omega)$ do not satisfy it. Therefore the nonlinear terms on the right hand side of the first order governing equation (3.43) do not resonate first order solution while linear terms do. To avoid unbounded resonance for η_{s1} we must have

$$2\eta_{s0,xX} - \frac{1}{3}u_{0,xxxt} + \frac{1}{2U_r}\eta_{b0,xxtt} = 0 \quad (3.54)$$

Taking one integration with respect to x and substituting from equation (3.53), we end up getting

$$\zeta_{,X} + \beta\zeta_{,\sigma\sigma\sigma} = 0 \quad (3.55)$$

$$\beta = \frac{1}{6} + \frac{1}{4U_r}(c^2 - 1) \quad (3.56)$$

Assuming a periodic solution both in time and space in the form

$$\zeta = \zeta_0 e^{i(k^*X + \omega^*\sigma)} \quad (3.57)$$

we get the dispersion relation for the envelope

$$k^* = \beta\omega^{*3} \quad (3.58)$$

For $c^2 < 1 - \frac{2U_r}{3}$, the envelope moves in the same direction that the original wave is moving while for higher values it moves in the opposite direction.

4 Conclusion

We presented formulation of a flexible (visco-elastic) sea floor carpet composed of mass of the carpet and vertically acting spring and generators with the latter modeled as a linear viscous damper. We showed that the coupled system of gravity waves and our carpet admits two modes of propagating waves: the surface-mode and the bottom-mode. The major difference between the two mode is that the rate of decay of a surface-mode wave is higher for longer waves, whereas for a bottom-mode shorter waves are damped faster. The idea presented here can, essentially, incorporate any mud model and its performance under different models/assumptions may worth further investigation. Specifically the performance may be substantially increased if frequency-dependent damping and stiffness coefficients are incorporated (see e.g. 16). While the discussion on the engineering aspects and implementation issues of the presented idea is beyond the scope of this manuscript, it was brought to our attention that a flexible membrane (floating) wave energy converter (named Lylipad), sharing most of its implementation aspects with our flexible carpet, is already under the investigation by the industry.

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